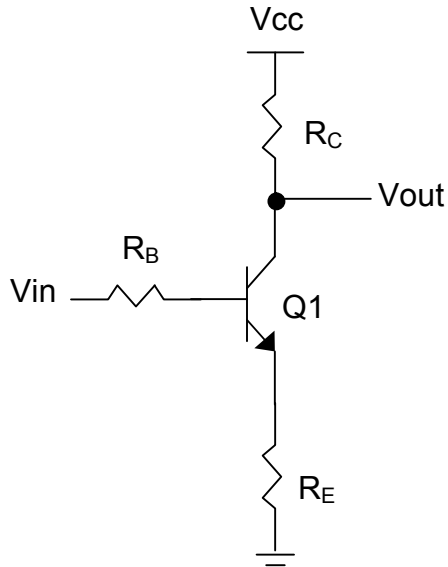


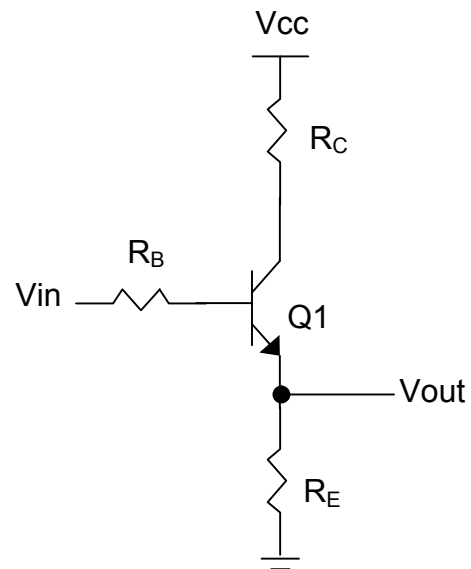
ECE 323 HW # 3

Prob. 1-3: Find the expression for small signal R_{in} , G_m , R_{out} and Gain for all the circuits. Assume all the BJTs are biased in forward active region and MOSFETs in saturation. Consider $r_o = \infty$

Prob.1



(a)



(b)

$$R_{in} = R_B + r_{\pi} + R_E(1+\beta)$$

$$R_{out} = R_C$$

$$G_m = \frac{-r_{\pi}}{R_{in}} g_m$$

$$Gain = G_m \cdot R_{out} = \frac{-\beta R_C}{R_B + r_{\pi} + R_E(1+\beta)}$$

$$R_{in} = R_B + r_{\pi} + R_E(1+\beta)$$

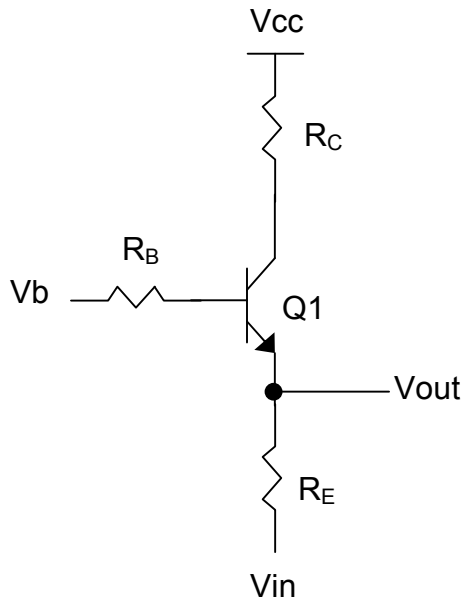
$$R_{out} = R_E \parallel \frac{R_B + r_{\pi}}{1+\beta}$$

$$G_m = \frac{1}{R_B + r_{\pi}} + \frac{r_{\pi}}{R_B + r_{\pi}} g_m = \frac{1+\beta}{R_B + r_{\pi}}$$

$$Gain = G_m \cdot R_{out}$$

but you can just apply voltage divider to get gain, even without knowing G_m & R_{out} .

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(c)

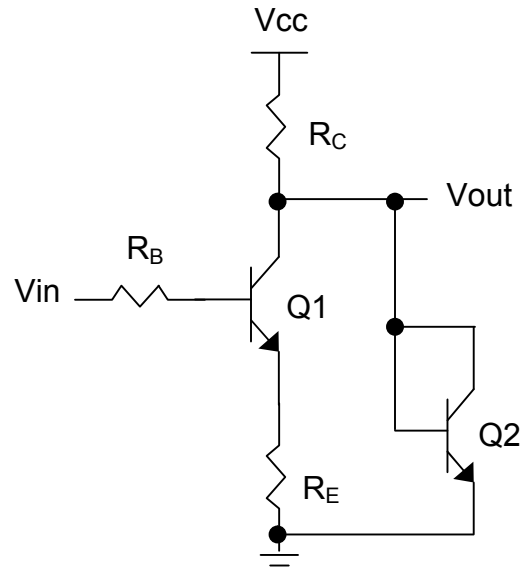
$$R_{in} = R_E + \frac{R_B + r_{\pi}}{1 + \beta}$$

$$R_{out} = R_E \parallel \frac{R_B + r_{\pi}}{1 + \beta}$$

$$G_m = \frac{1}{R_E}$$

$$G_{in} = G_m \cdot R_{out}$$

or just apply voltage divider



(d)

$$R_{in} = R_B + r_{\pi 1} + R_E (1 + \beta_1)$$

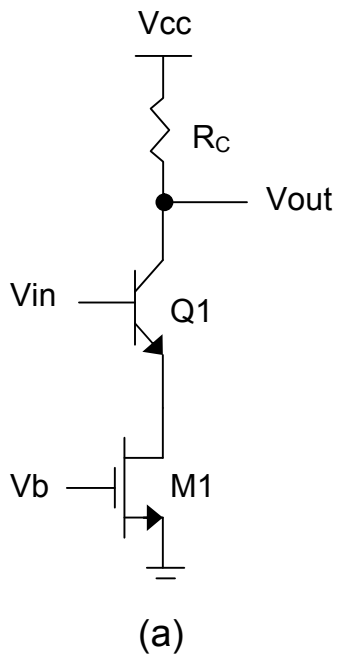
$$R_{out} = \frac{r_{\pi 2}}{1 + \beta_2} \parallel R_C$$

$$\approx \frac{1}{g_{m2}}$$

$$G_m = \frac{-r_{\pi 1} \cdot g_{m1}}{R_B + r_{\pi 1} + R_E (1 + \beta_1)}$$

$$G_{in} = G_m \cdot R_{out}$$

Prob.2



$$r_{o(M1)} = \infty$$

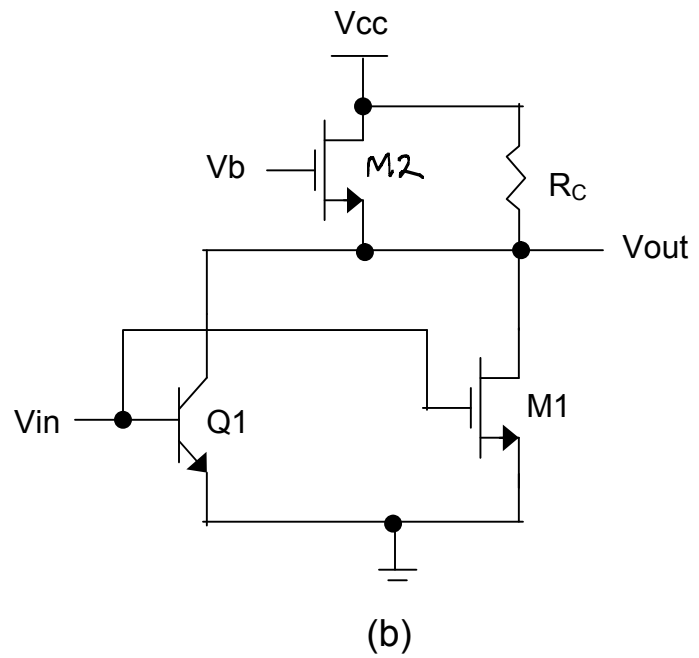


$$R_{in} = \infty$$

$$G_m = 0$$

$$G_{in} = 0$$

$$R_{out} = R_c$$



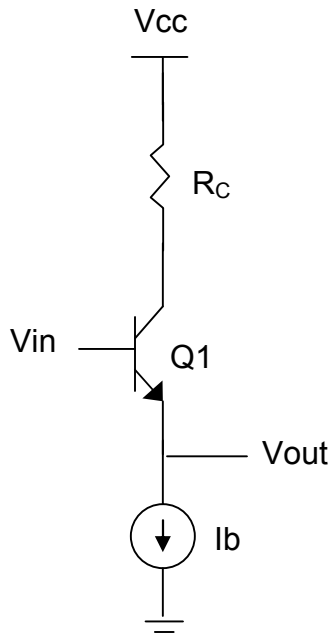
$$R_{in} = r_{\pi 1}$$

$$R_{out} = R_c \parallel \frac{1}{g_{m2}}$$

$$G_m = (g_{mQ1} + g_{mM1})$$

$$G_{in} = G_m \cdot R_{out}$$

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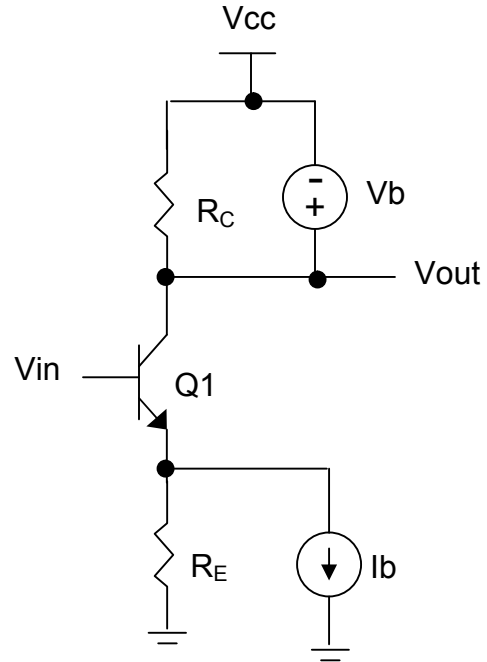
(c)

$$R_{in} = \infty \text{ (current source)}$$

$$R_{out} = \frac{r_{\pi}}{1+\beta}$$

$$G_m = \frac{1}{r_{\pi}} + g_m = \frac{1+\beta}{r_{\pi}}$$

$$Gain = G_m \cdot R_{out} = 1$$



(d)

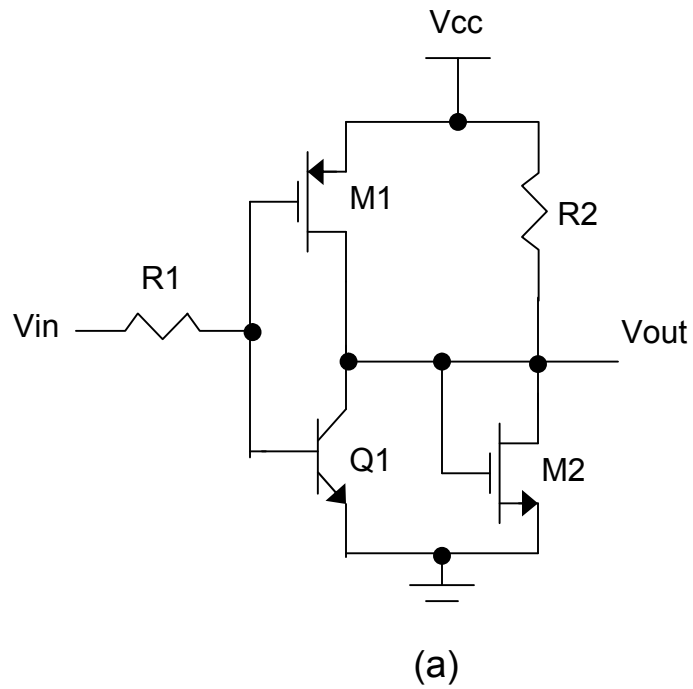
$$R_{in} = r_{\pi} + R_E(1+\beta)$$

$$R_{out} = 0 \text{ (voltage source)}$$

$$G_m = \frac{-r_{\pi} \cdot g_m}{r_{\pi} + R_E(1+\beta)}$$

$$Gain = G_m \cdot R_{out} = 0$$

Prob. 3



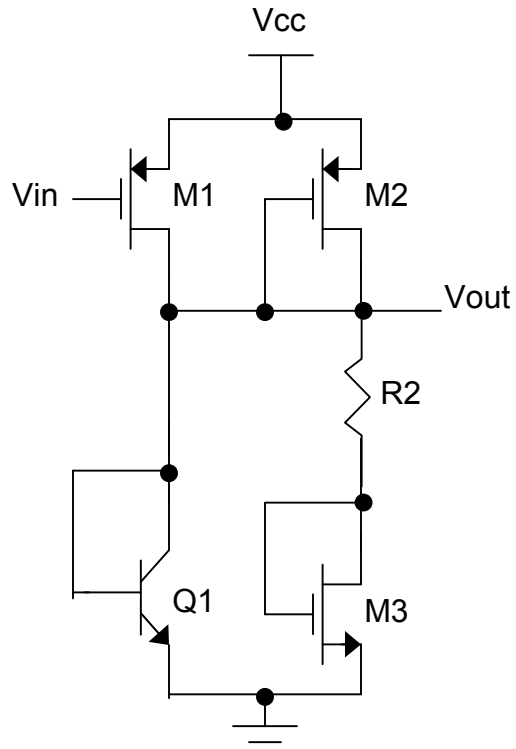
$$R_{in} = R_1 + r_{\pi}$$

$$R_{out} = R_2 \parallel \frac{1}{g_{m2}}$$

$$G_m = \frac{-r_{\pi}}{R_1 + r_{\pi}} (g_{m_{Q1}} + g_{m_{M1}})$$

$$Gain = G_m \cdot R_{out}$$

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(b)

$$Z_{in} = \infty$$

$$R_{out} = \frac{1}{g_{m2}} \parallel \left(R_2 + \frac{1}{g_{m3}} \right) \parallel \frac{r_{\pi}}{1+\beta}$$

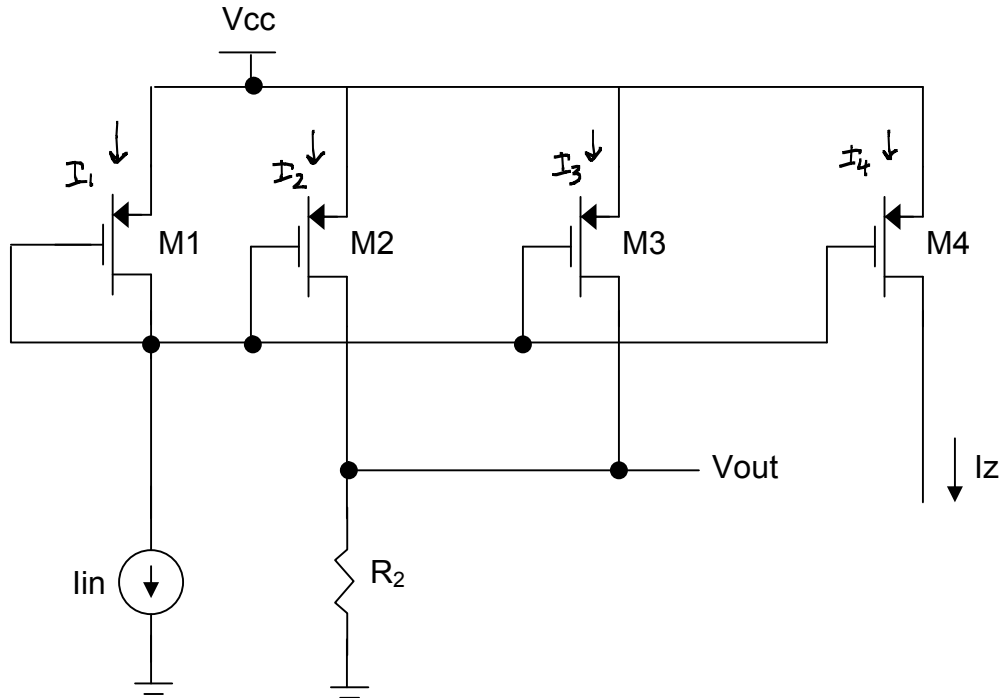
$$G_m = -g_{mM1}$$

$$G_{ain} = G_m \cdot R_{out}$$

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Prob. 4

For the circuit given below, find the expression for output resistance R_{out} , transimpedance V_{out}/I_{in} and current gain I_z/I_{in} . Assume all the MOSFETs having same W/L and operating in saturation region. Consider $r_o = \infty$



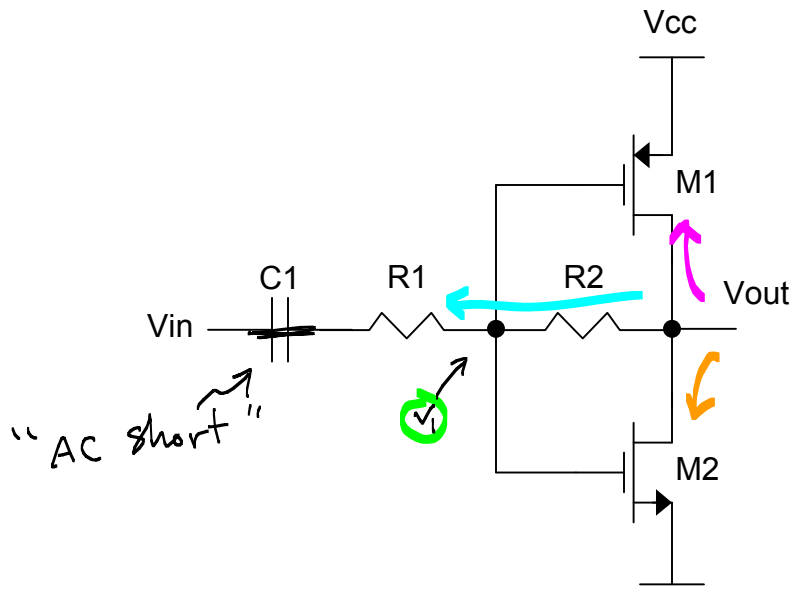
$$\text{all same } \frac{W}{L} \rightarrow I_{in} = I_1 = I_2 = I_3 = I_4 = I_z \rightarrow \frac{I_z}{I_{in}} = 1$$

$$R_{out} = R_2 \quad (r_o = \infty)$$

$$\frac{V_{out}}{I_{in}} = 2 \cdot R_2$$

Prob. 5

For the circuit given below, find the expression for R_{in} , R_{out} , G_m and gain. Assume all the MOSFETs operating in saturation region and consider $r_o = \infty$. Plot the behavior of output gain vs R_2 .



V_1 controls V_{gs} of $M1$ and $M2$, in parallel.

$$R_{in} = R_1 + \frac{1}{g_{m1} + g_{m2}}$$

$$R_{out} = (R_1 + R_2) \parallel \left[\frac{R_1}{R_1 + R_2} (g_{m1} + g_{m2}) \right]^{-1} = \frac{R_1 + R_2}{R_1 (g_{m1} + g_{m2}) + 1}$$

$(V_{in} = 0)$

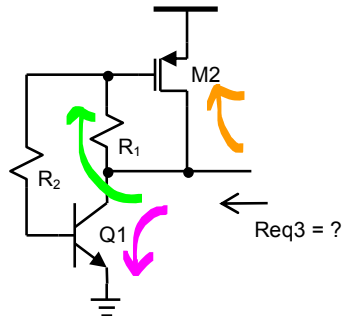
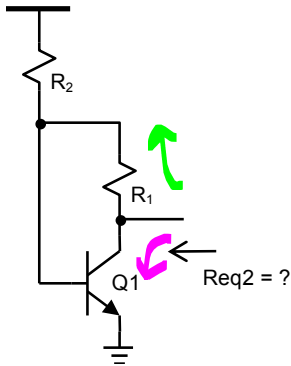
$$G_m = \frac{1}{R_1 + R_2} - \frac{R_2}{R_1 + R_2} (g_{m1} + g_{m2}) = \frac{1 - R_2 (g_{m1} + g_{m2})}{R_1 + R_2}$$

$(V_{out} \text{ shorted})$

$$Gain = G_m \cdot R_{out} = \frac{1 - R_2 (g_{m1} + g_{m2})}{1 + R_1 (g_{m1} + g_{m2})}$$

Prob. 6

Find Req2 and Req3

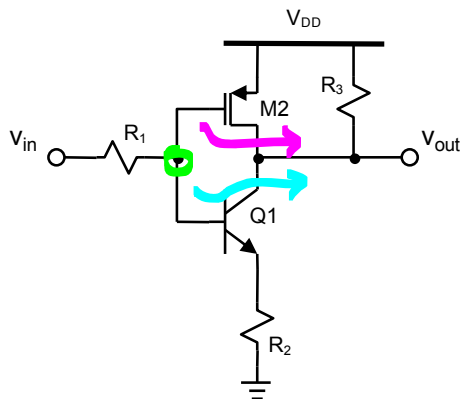


$$Req2 = \left[R1 + (R2 || r_{\pi 1}) \right] || \left[\frac{(R2 || r_{\pi 1}) \cdot g_{m1}}{R1 + (R2 || r_{\pi 1})} \right]^{-1}$$

$$Req3 = \left(R1 + R2 + r_{\pi 1} \right) || \left(\frac{r_{\pi 1} \cdot g_{m1}}{R1 + R2 + r_{\pi 1}} \right)^{-1} || \left(\frac{(R2 + r_{\pi 1}) \cdot g_{m2}}{R1 + R2 + r_{\pi 1}} \right)^{-1}$$

Prob. 7

Find the input-to-output small-signal gain.



$$Gain = \frac{r_{\pi 1} + R_2(1 + \beta_1)}{R_1 + r_{\pi 1} + R_2(1 + \beta_1)} \left(\frac{r_{\pi 1} \cdot g_{m1}}{r_{\pi 1} + R_2(1 + \beta_1)} + g_{m2} \right) \cdot R_3$$