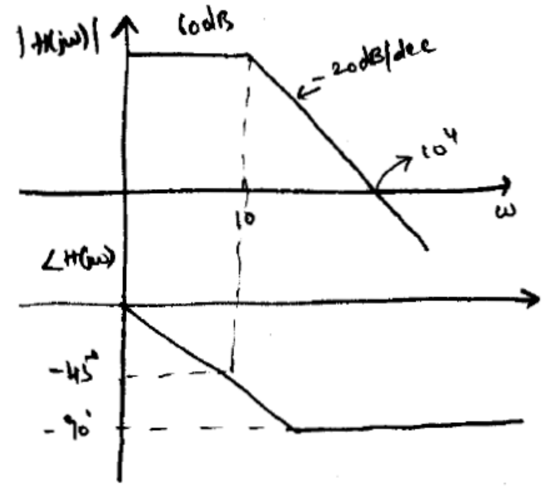
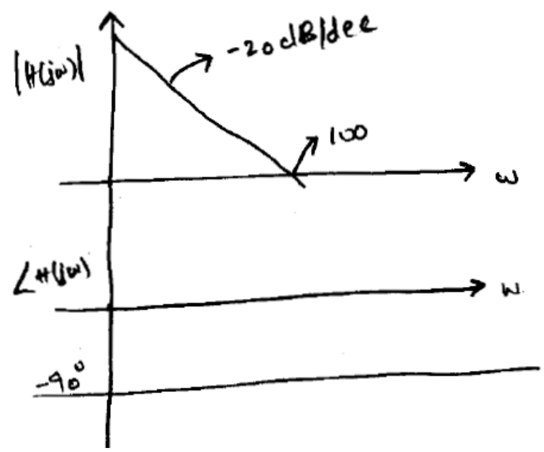
ECE 323
Homework 4

1. Sketch the magnitude and phase response of the following transfer functions.

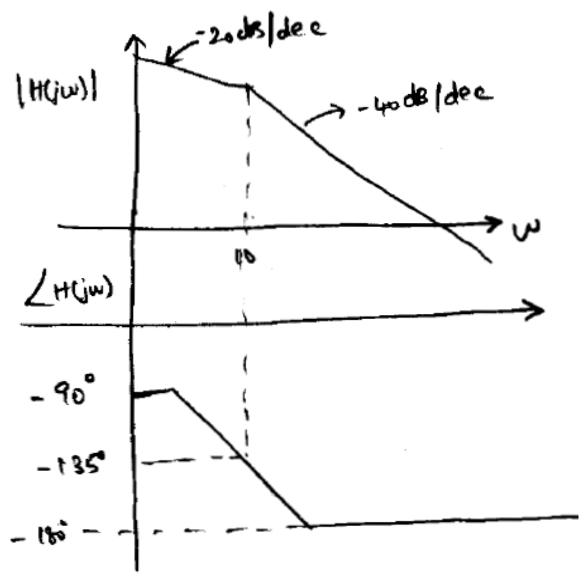
(a) $H(s) = \dfrac{1000}{\left(1+\dfrac{s}{10}\right)}$



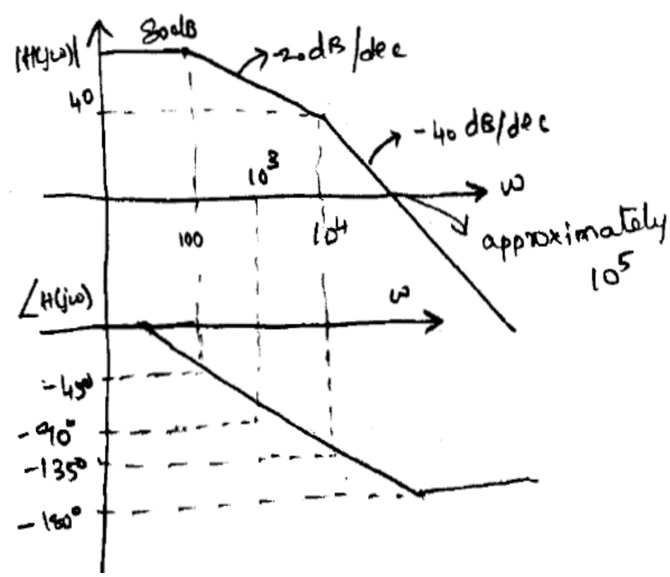
(b) $H(s) = \dfrac{100}{s}$



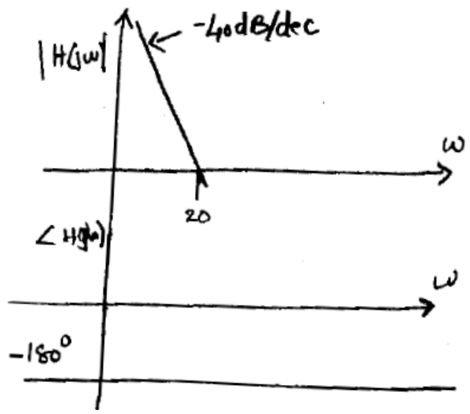
(c) $H(s) = \dfrac{10000}{s(s+10)}$



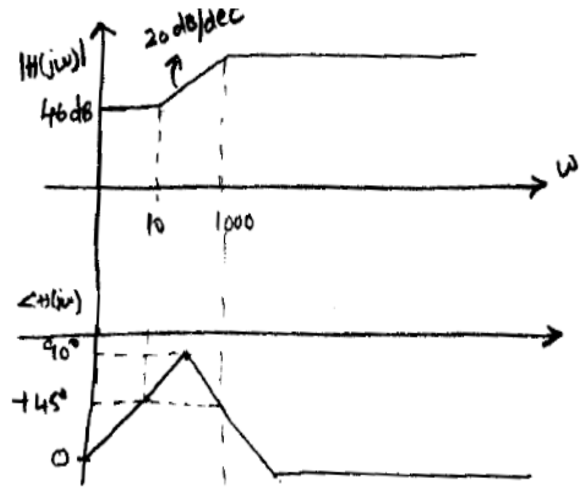
(d) $H(s) = \dfrac{10000}{\left(1+\dfrac{s}{100}\right)\left(1+\dfrac{s}{10^4}\right)}$



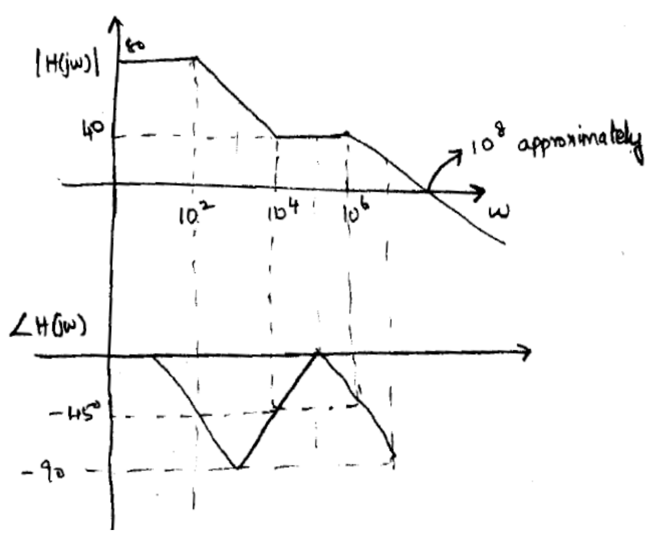
(e) $H(s)=\dfrac{400}{s^2}$



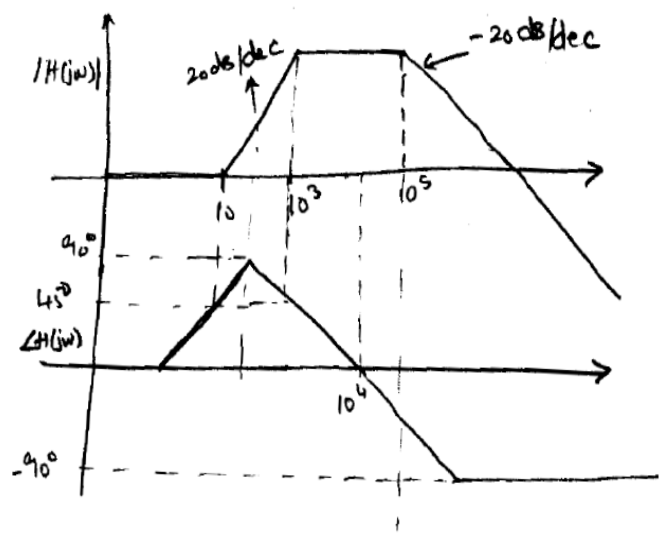
(f) $H(s)=\dfrac{200\left(1+\dfrac{s}{10}\right)}{\left(1+\dfrac{s}{10^3}\right)}$



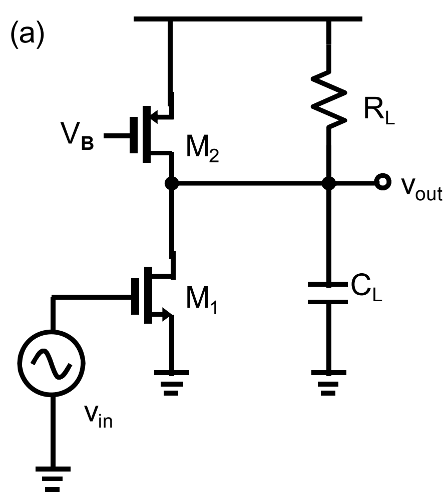
(g) $H(s)=\dfrac{10^4\left(1+\dfrac{s}{10^4}\right)}{\left(1+\dfrac{s}{100}\right)\left(1+\dfrac{s}{10^6}\right)}$

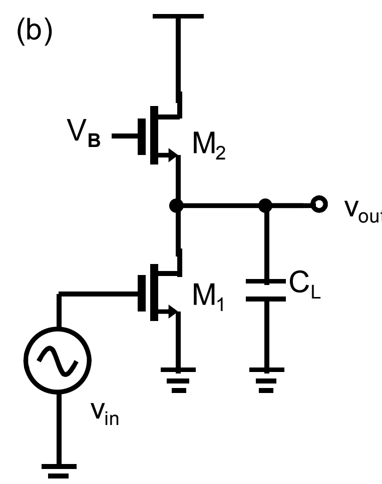


(h) $H(s)=\dfrac{\left(1+\dfrac{s}{10}\right)}{\left(1+\dfrac{s}{10^3}\right)\left(1+\dfrac{s}{10^5}\right)}$



2. Find the input to output transfer function H(s). In other words, find the low frequency small signal gain and poles. Ignore all intrinsic capacitances. Assume $r_o = \infty$.





$\text{Gain} = -g_{m1} R_L$

$\omega_p = \dfrac{1}{C_L \cdot R_L}$

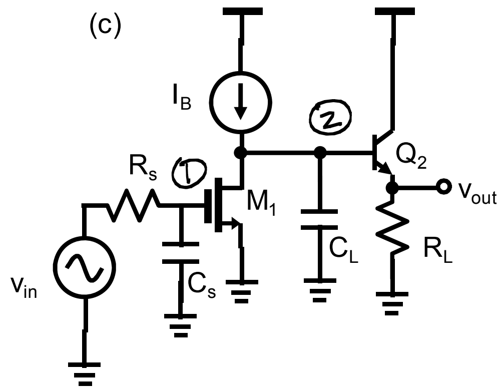
$H(s) = \dfrac{\text{Gain}}{1 + \frac{s}{\omega_p}}$

$\text{Gain} = -g_{m1} \cdot \dfrac{1}{g_{m2}}$

$\omega_p = \dfrac{1}{C_L \cdot \frac{1}{g_{m2}}}$

$H(s) = \dfrac{\text{Gain}}{1 + \frac{s}{\omega_p}}$

(c)



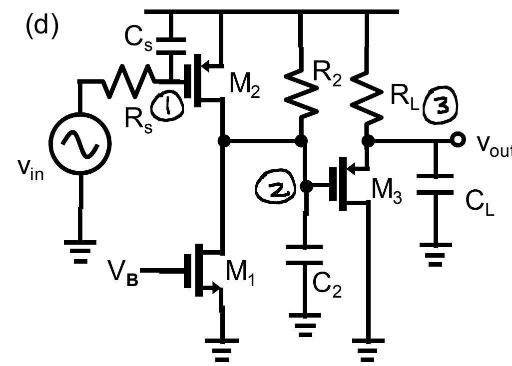
$$\text{Gain} = -g_{m1} \cdot \left[r_{\pi 2} + R_L(1+\beta)\right] \cdot \frac{R_L}{\frac{r_\pi}{1+\beta} + R_L}$$

$$\omega_1 = \frac{1}{C_s \cdot R_s}$$

$$\omega_2 = \frac{1}{C_L \cdot \left[r_{\pi 2} + R_L(1+\beta)\right]}$$

$$H(s) = \frac{\text{Gain}}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)}$$

(d)



$$\text{Gain} = -g_{m2} R_2 \cdot \frac{R_L}{\frac{1}{g_{m3}} + R_L}$$

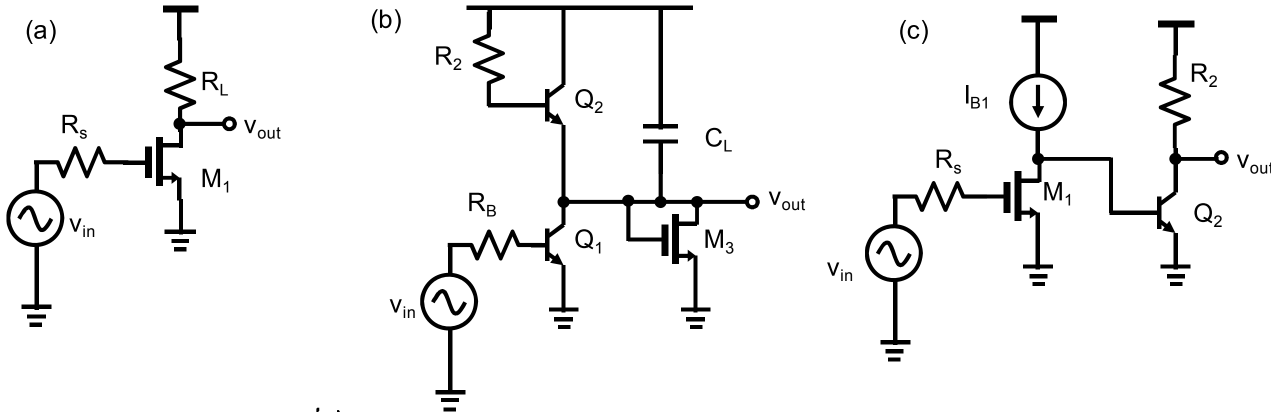
$$\omega_1 = \frac{1}{C_s \cdot R_s}$$

$$\omega_2 = \frac{1}{C_2 \cdot R_2}$$

$$\omega_3 = \frac{1}{C_L \cdot \left(R_L \,\|\, \frac{1}{g_{m3}}\right)}$$

$$H(s) = \frac{\text{Gain}}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_3}\right)}$$

3. Find the approximate -3dB bandwidth (corner frequency) for each of the following circuits using zero-value time constant analysis. Include intrinsic capacitances and assume $r_o = \infty$.



(a)

$$\omega_{3dB} = \frac{1}{C_{gs}\cdot R_S + C_{gd}\cdot \text{"}R_{gd}\text{"}}$$

$$\text{"}R_{gd}\text{"} = R_S\,(1 + g_m R_L) + R_L$$

(b)

$$\omega_{3dB} = \frac{1}{C_{\pi 1}\text{"}R_{\pi 1}\text{"} + C_{\mu 1}\text{"}R_{\mu 1}\text{"} + C_{\pi 2}\text{"}R_{\pi 2}\text{"} + C_{\mu 2}\text{"}R_{\mu 2}\text{"} + (C_{gs3} + C_L)\,R_{out}}$$

$$\text{"}R_{\pi 1}\text{"} = R_B \,||\, r_{\pi 1}$$

$$\text{"}R_{\mu 1}\text{"} = (R_B \,||\, r_{\pi 1})(1 + g_{m1} R_{out}) + R_{out}$$

$$\text{"}R_{\pi 2}\text{"} = \frac{R_2 + \frac{1}{g_{m3}}}{1 + g_{m2}\frac{1}{g_{m3}}} \,||\, r_{\pi 2}$$

$$\text{"}R_{\mu 2}\text{"} = R_2 \,||\, \left[r_\pi + \frac{1}{g_{m3}}(1+\beta_2)\right]$$

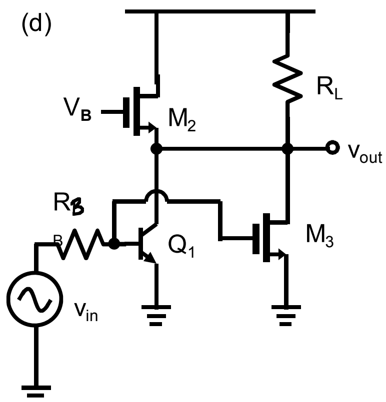
$$R_{out} = \frac{1}{g_{m3}} \,||\, \frac{R_2 + r_{\pi 2}}{1+\beta_2}$$

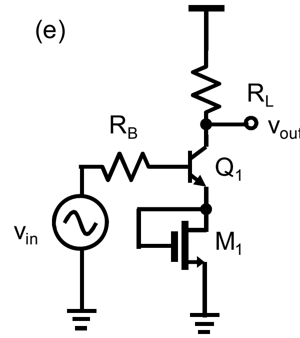
(c)

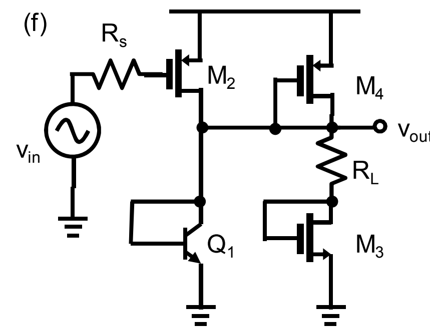
$$\omega_{3dB} = \frac{1}{C_{gs1} R_S + C_{gd1}\text{"}R_{gd1}\text{"} + C_{\pi 2} r_{\pi 2} + C_{\mu 2}\text{"}R_{\mu 2}\text{"}}$$

$$\text{"}R_{gd1}\text{"} = R_S(1 + g_{m1} r_{\pi 2}) + r_{\pi 2}$$

$$\text{"}R_{\mu 2}\text{"} = r_{\pi 2}(1 + g_{m2} R_2) + R_2$$







(d)

$$\omega_{3dB} = \frac{1}{(C_{\pi1}+C_{gs3})``R_{\pi1}" + (C_{\mu1}+C_{gd3})``R_{\mu1}" + C_{gs2}\cdot R_{out}}$$

$$``R_{\pi1}" = R_B \| r_{\pi1}$$

$$``R_{\mu1}" = (R_B \| r_{\pi1})\left(1+[g_{m1}+g_{m3}]\,R_{out}\right) + R_{out}$$

$$R_{out} = \frac{1}{g_{m2}} \| R_L$$

(e)

$$\omega_{3dB} = \frac{1}{C_{\pi1}``R_{\pi1}" + C_{\mu1}``R_{\mu1}" + C_{gs1}\cdot ``R_E"}$$

$$``R_{\pi1}" = \frac{R_B + \frac{1}{g_{mM1}}}{1+g_{mQ1}\cdot\frac{1}{g_{mM1}}} \Bigg\| r_{\pi1}$$

$$``R_{\mu1}" = R_B \Big\| \left[r_{\pi1} + \frac{1}{g_{mM1}}\cdot(1+\beta)\right]\left(1+\frac{r_{\pi1}\cdot g_{mQ1}R_L}{r_{\pi1}+(1+\beta)\frac{1}{g_{mM1}}}\right) + R_L$$

$$``R_E" = \frac{1}{g_{mM1}} \Big\| \frac{R_B + r_{\pi1}}{1+\beta}$$

(f)

$$\omega_{3dB} = \frac{1}{(C_{\pi1}+C_{gs4})R_{out} + C_{gs3}``R_{gs3}" + C_{gs2}``R_{gs2}" + C_{gd2}``R_{gd2}"}$$

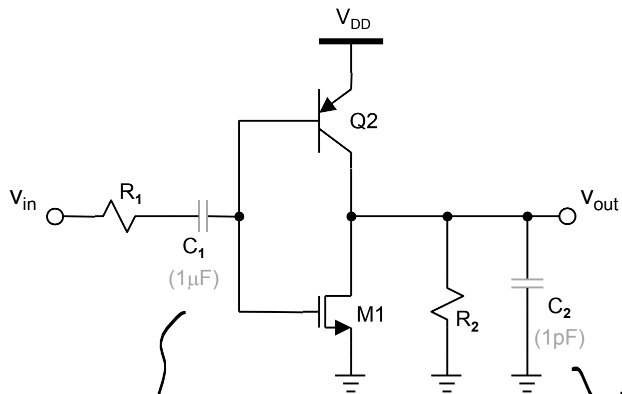
$$R_{out} = \frac{r_{\pi1}}{1+\beta} \Big\| \frac{1}{g_{m4}} \Big\| \left[R_L + \frac{1}{g_{m3}}\right]$$

$$``R_{gs3}" = \frac{1}{g_{m3}} \Big\| \left[R_L + \left(\frac{1}{g_{m4}} \Big\| \frac{r_{\pi1}}{1+\beta}\right)\right]$$

$$``R_{gs2}" = R_s$$

$$``R_{gd2}" = R_s(1+g_{m2}R_{out}) + R_{out}$$

4. Find the input-to-output transfer function H(s).

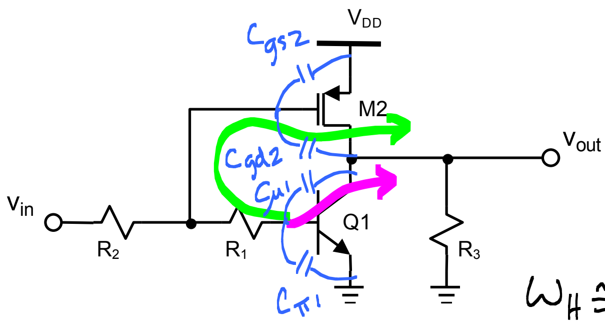


"high pass" at low frequency

"low pass" at high frequency

$$H(s) = \frac{-r_{\pi 2}}{R_1 + r_{\pi 2}} \left(g_{m1} + g_{m2} \right) R_2 \cdot \frac{s}{s + \frac{1}{C_1 \cdot [R_1 + r_{\pi 2}]}} \cdot \frac{1}{1 + s C_2 R_2}$$

5. Find the “upper 3dB frequency” ω_H (considering the intrinsic capacitances).



$$\omega_H \cong \frac{1}{C_{\pi 1}\,``R_{\pi 1}" + C_{\mu 1}\,``R_{\mu 1}" + C_{gs2}\,``R_{gs2}" + C_{gd2}\,``R_{gd2}"}$$

$$``R_{\pi 1}" = r_{\pi 1} \,\|\, (R_1 + R_2)$$

$$``R_{\mu 1}" = ``R_{\pi 1}" \left(1 + \left(\underline{g_{m1}} + \underline{\frac{R_2}{R_1+R_2} g_{m2}}\right) R_3\right) + R_3$$

$$``R_{gs2}" = R_2 \,\|\, (R_1 + r_{\pi 1})$$

$$``R_{gd2}" = ``R_{gs2}" \left(1 + \left(g_{m2} + \frac{r_{\pi 1}}{R_1 + r_{\pi 1}} g_{m1}\right) R_3\right) + R_3$$