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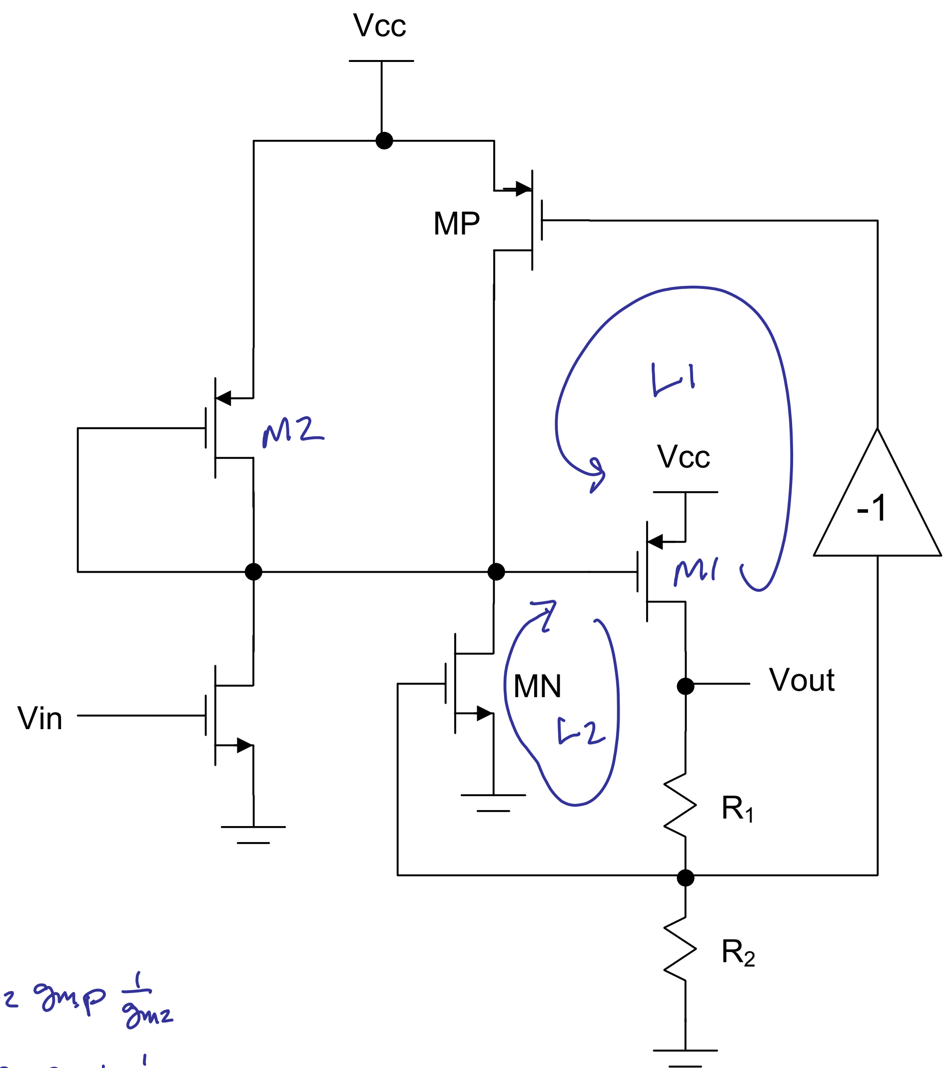
Prob-1. Determine in which of the two cases (a) $g_{mp} > g_{mn}$ (b) $g_{mp} < g_{mn}$ the circuit will be unstable and why? g_{mp} and g_{mn} are respective transconductances of MP and MN.



$L_1 = -g_{m1} R_2 g_{mp} \frac{1}{g_{m2}}$

$L_2 = + g_{m1} R_2 g_{mN} \frac{1}{g_{m2}}$

($g_{mp} > g_{mN}$) yields net negative feedback loop → STABLE

($g_{mp} < g_{mN}$) yields net positive feedback loop → UNSTABLE

(Only if magnitude of loopgain > 1.)

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Prob-2. A second order system has an open loop transfer function $H(s) = \dfrac{2000}{(\frac{s}{10}+1)(\frac{s}{p}+1)}$ and feedback factor of β.

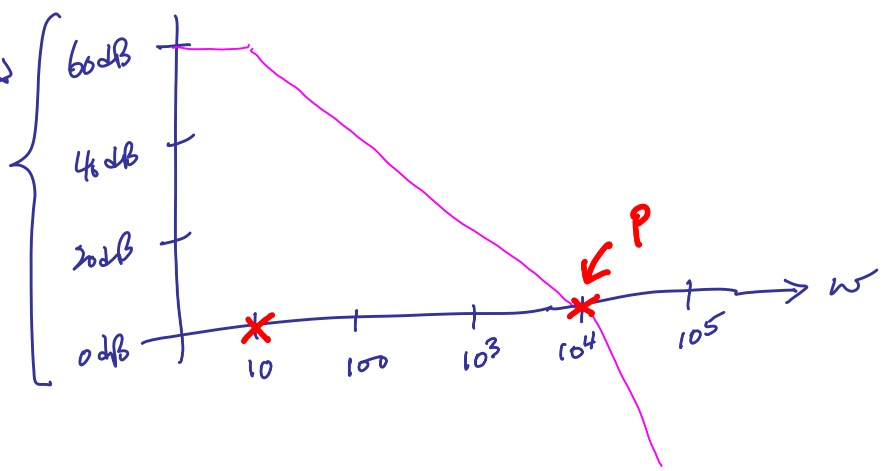
Determine the location of pole p for which the loop phase margin is $45°$ if (a) $\beta=1/2$ and (b) $\beta=1/20$. Show the bode plot of loop transfer function for both the cases.

(*) 45° phase margin implies 2nd pole is located at 0dB loopgain.

$$L(s) = H(s)\cdot\beta$$

$$L(s)\Big|_{\beta=1/2} = \frac{1000}{(1+\frac{s}{10})(1+\frac{s}{p})} \longrightarrow \text{want } p=10^4 \text{ for } 45° \text{ P.M.}$$

$$L(s)\Big|_{\beta=\frac{1}{20}} = \frac{100}{(1+\frac{s}{10})(1+\frac{s}{p})} \longrightarrow \text{want } p=10^3 \text{ for } 45° \text{ PM}$$



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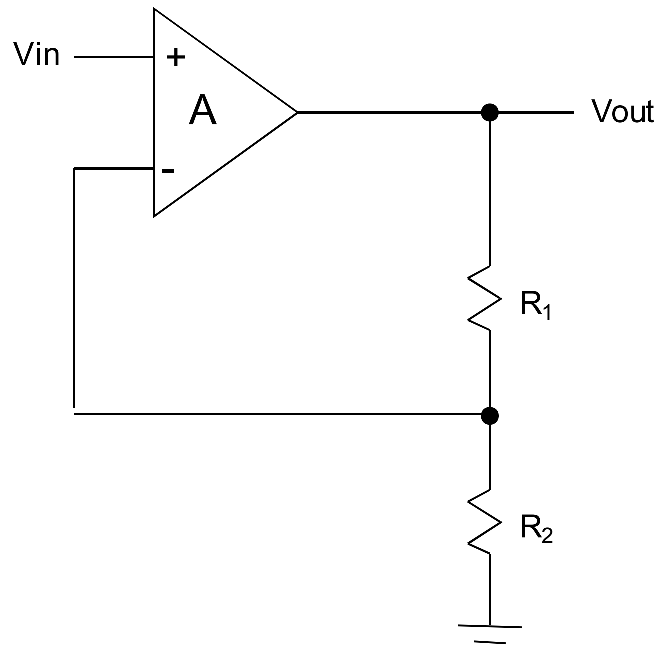
Prob-3. Figure below shows a non-inverting amplifier achieved by connecting an opamp in negative feedback configuration. Assuming open loop gain of opamp $A=\infty$, the closed loop gain *Vout/Vin* of the circuit is given by $1+R1/R2$ which is simply $1/\beta$ where β is the feedback factor $R2/(R1+R2)$. Find the expression for closed loop gain if A is finite and determine % error in the closed loop gain for (a) $A=1000$ and (b) $A=10000$.



$$\beta = \frac{R_2}{R_1+R_2}$$

$$\text{Closed loop gain} = \frac{V_{out}}{V_{in}} = \frac{A}{1+A\beta} = \underbrace{\frac{1}{\beta}}_{\text{ideal gain} \sim \frac{R_1+R_2}{R_2}} \frac{A\beta}{1+A\beta}$$

$$\text{Error} = 1 - \frac{A\beta}{1+A\beta} = \frac{1}{1+A\beta}$$

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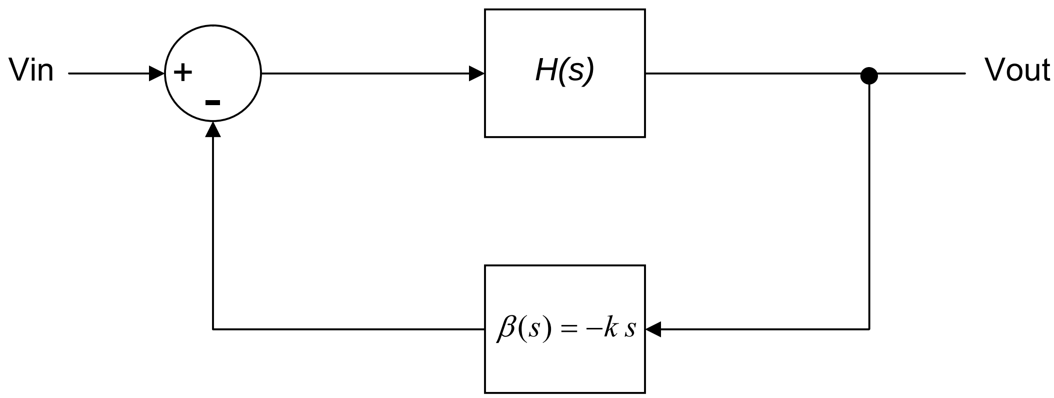
Prob-4. A system is said to be unstable if any of the poles lie in right half of the s-plane. This property can be used to convert a second order system into oscillator by using negative feedback and choosing a proper feedback function. Find the closed loop transfer function *Vout(s)/Vin(s)* of the system and show how the location of poles change w.r.t *k* in s-plane.



$$H(s) = \frac{A}{s^2 + \frac{w_n}{Q_0}s + w_n^{\ 2}}$$

$$A(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{H(s)}{1 - H(s)\cdot ks} = \frac{A}{s^2 + s\underbrace{\left(\frac{w_n}{Q_0} - Ak\right)}_{\text{"b" part of quadratic formula}} + w_n^2}$$

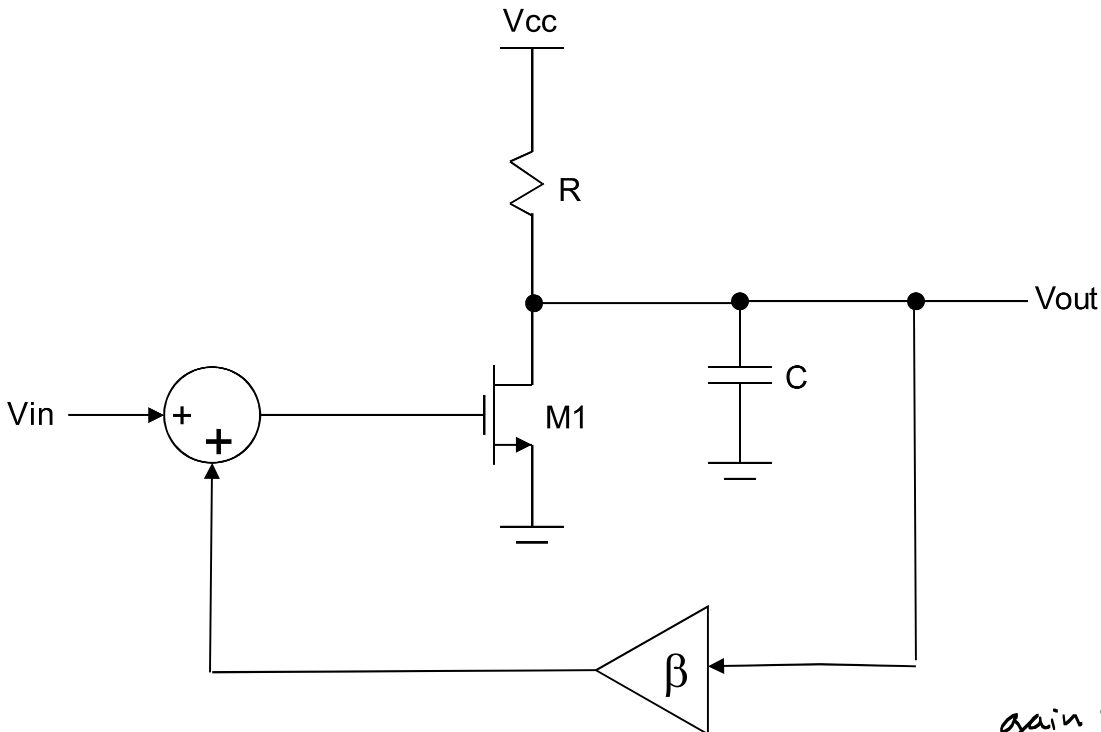
$$s = \frac{-b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

$(b=0) \rightarrow k = \frac{w_n}{Q_0 A} \rightarrow$ poles on $j\omega$-axis (all imaginary)

$(b>0) \rightarrow k < \frac{w_n}{Q_0 A} \rightarrow$ STABLE (poles on left half plane)

$(b<0) \rightarrow k > \frac{w_n}{Q_0 A} \rightarrow$ UNSTABLE

Prob-5. A common source amplifier with DC gain *gm·R* and pole location *wp=1/RC* is connected in negative feedback configuration as shown in figure below. Determine the closed loop transfer function *Vout(s)/Vin(s)*, closed loop DC gain and closed loop *-3dB* bandwidth of the circuit. Show the bode plot for gain and phase for (a) β=0 (b) β=1 and (c) β=1/2 and compare the DC gain and Bandwidth of the closed system with that of open loop gain and bandwidth of common source amplifier. Consider $ro=\infty$.



$$A(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{-g_{m1}R\,\frac{1}{1+sRC}}{1+g_{m}R\frac{1}{1+sRC}\cdot\beta} = \frac{\left(\frac{-g_{m1}R}{1+g_{m1}R\beta}\right)}{1+\frac{sRC}{(1+g_{m1}R\beta)}}$$

gain $= \frac{-g_{m1}R}{1+g_{m1}R\beta}$

$BW = \frac{1}{RC}(1+g_{m1}R\beta)$

When $\beta=0$, A(s) is same as openloop.

gain $= -g_{m1}R$

$BW = \frac{1}{RC} =$ pole

For $\beta=\frac{1}{2}$, $\beta=1$,

A(s) sees decrease in gain and increase in BW, by $(1+g_{m1}R\beta)$.

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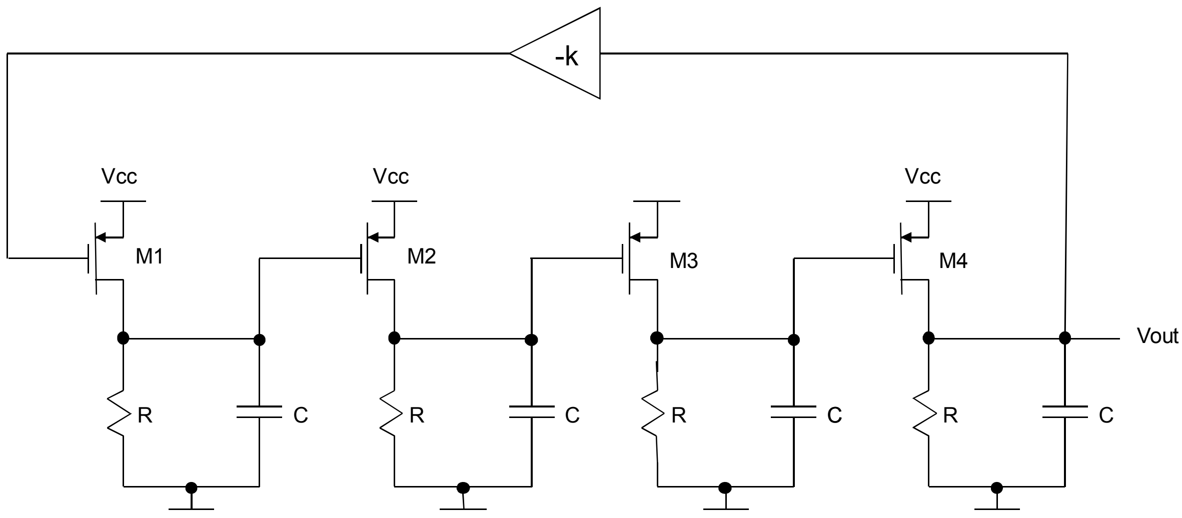
Prob-6 For the circuit shown below, find (a) minimum gain *k* if *R=1KΩ* and (b) minimum value of *R* if *k=0.25* for which the circuit will oscillate and determine the expression for frequency of oscillation in both the cases. Consider *ro=∞* and *gm=1m A/V* for all the PMOS.



Each stage to contribute 45° phase delay → results in positive FB loop.

45° is at pole frequency, where gain is reduced by $\sqrt{2}$ (3dB).

Loopgain at $\underbrace{\text{that frequency}}_{\frac{1}{RC}}$ $= \dfrac{k\,(g_{m1}R)^4}{(\sqrt{2})^4}$

$(R = 1k\Omega) \rightarrow$ want loopgain $k\left(\dfrac{g_{m1}R}{\sqrt{2}}\right)^4 > 1 \rightarrow k > \left(\dfrac{\sqrt{2}}{g_{m1}R}\right)^4 = 4$

$(k = 0.25) \rightarrow k\left(\dfrac{g_{m1}R}{\sqrt{2}}\right)^4 > 1 \longrightarrow R > \left(\dfrac{\sqrt{2}}{g_{m1}}\right)\cdot k^{-1/4} = 2\,k\Omega$

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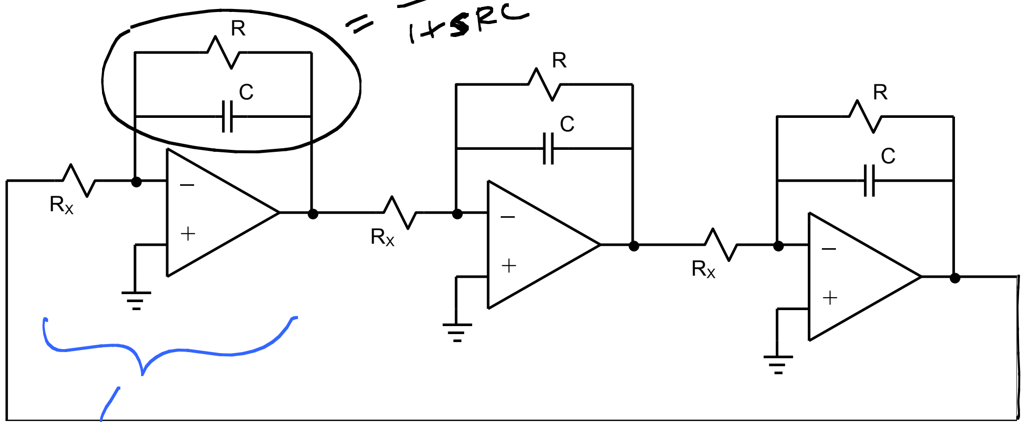
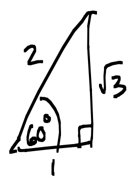
Prob-7. For the linear oscillator shown, find the oscillation frequency ω_o and R_X value required to ensure oscillation.

$$= \frac{R}{1+sRC}$$



$$-\frac{R}{R_X}\cdot\underbrace{\frac{1}{1+sRC}}_{\text{need } 60^\circ \text{ phase shift}} \longrightarrow \boxed{\omega_{osc} = \frac{\sqrt{3}}{RC}}$$



$$\left|\text{Loopgain}(s=j\omega_{osc})\right| = \left(\frac{R}{R_X}\,\frac{1}{2}\right)^3 \geq 1$$ to ensure oscillation

$$\longrightarrow \boxed{R_X \leq \frac{1}{2}R}$$

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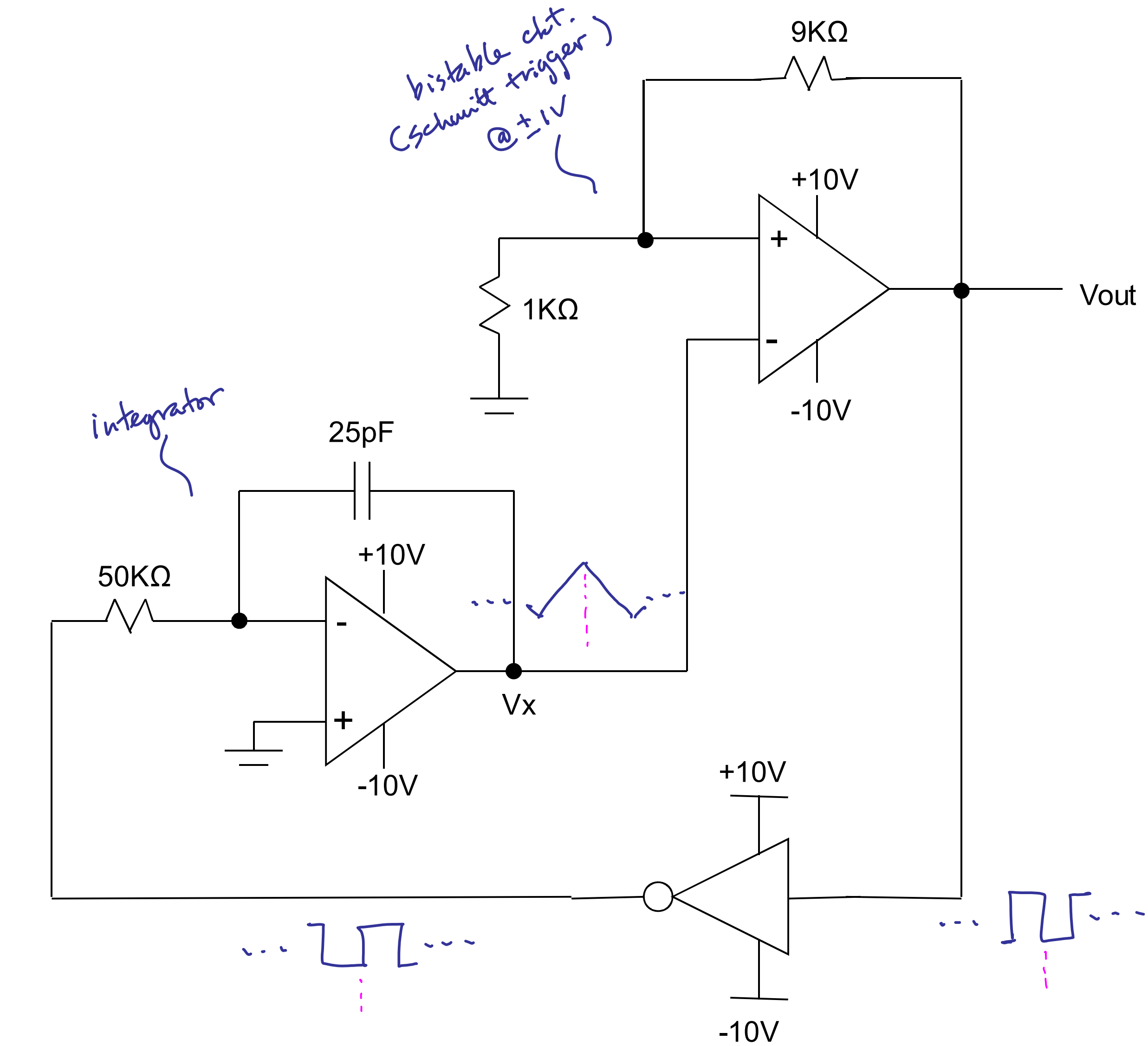
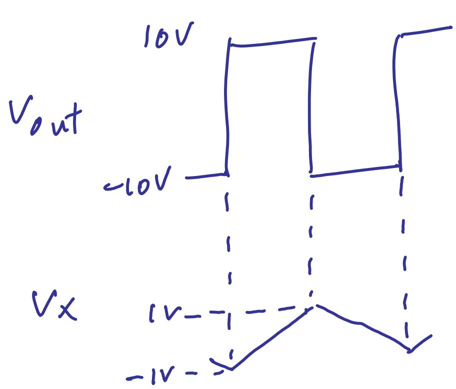
Prob-8. Plot the waveforms at Vout and Vx. Mark the voltages clearly and determine the frequency of oscillation.





$$I = C\frac{dV}{dt} \rightarrow \text{slope} = \frac{\Delta V}{\Delta t} = \frac{I}{C} = \frac{10}{RC} = \frac{10}{(50k\Omega)(25pF)}$$

$$T = \left(\frac{2}{\text{slope}}\right)\cdot 2 = 0.5\,\text{usec} \rightarrow f_{osc} = 2MHz$$

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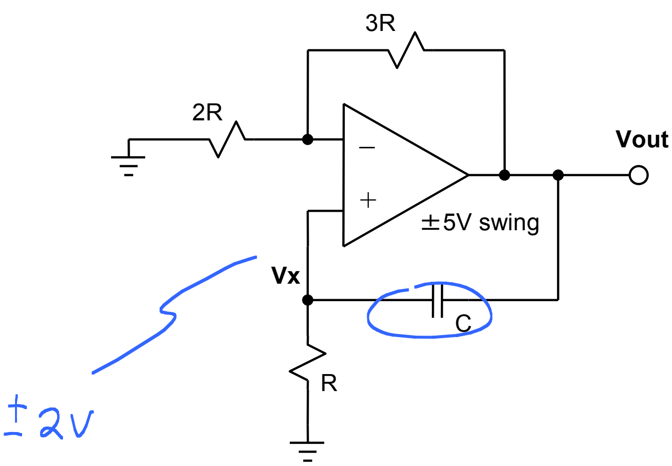
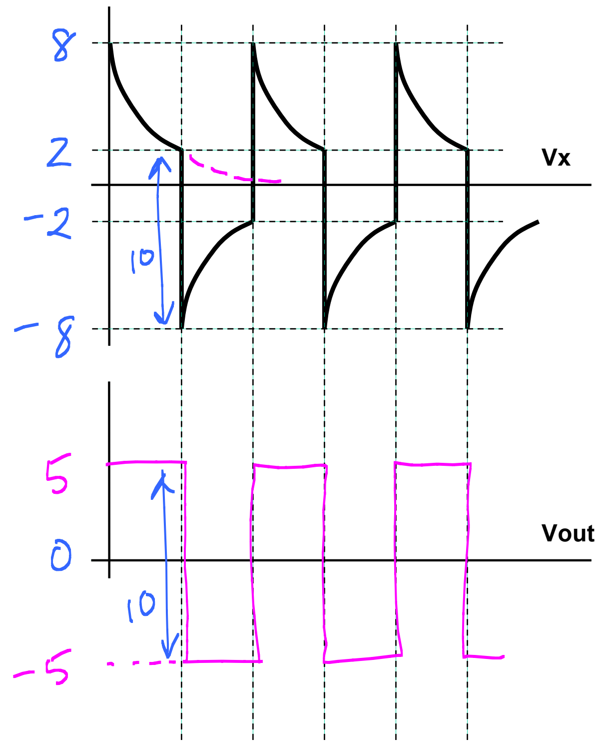
Prob-9. Note all critical voltage levels in the **Vx** waveform. Sketch **Vout** waveform, with proper time alignment and voltages. Find the resulting period (T_{osc}) of oscillation.





$$\frac{2}{8} = e^{-t_1/RC} \longrightarrow t_1 = RC\ln(4)$$

$$T_{osc} = 2t_1$$