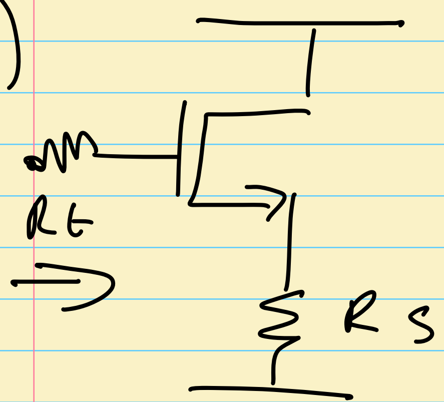
HW 2.

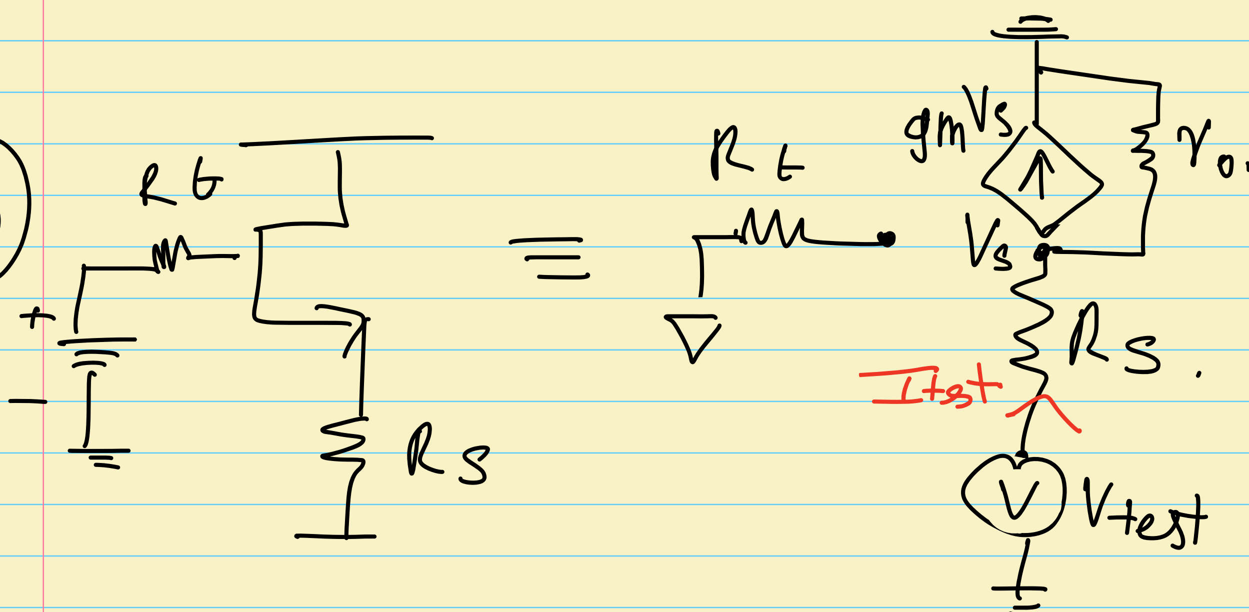
1) a)



$R_{in} = \infty$

Reason: gate is open ckt.

b)



$$I_{test} = \frac{V_{test} - V_s}{R_s} = g_m V_s + \frac{V_s}{r_o}.$$

$$V_{test} - V_s = g_m V_s R_s + \frac{V_s R_s}{r_o}.$$

$$V_{test} = V_s \left[1 + g_m R_s + \frac{R_s}{r_o} \right]$$

$$I_{test} = V_{test} - \frac{V_{test}}{\left[1 + g_m R_s + R_s / r_o \right]}$$

$$\frac{\quad}{R_S}.$$

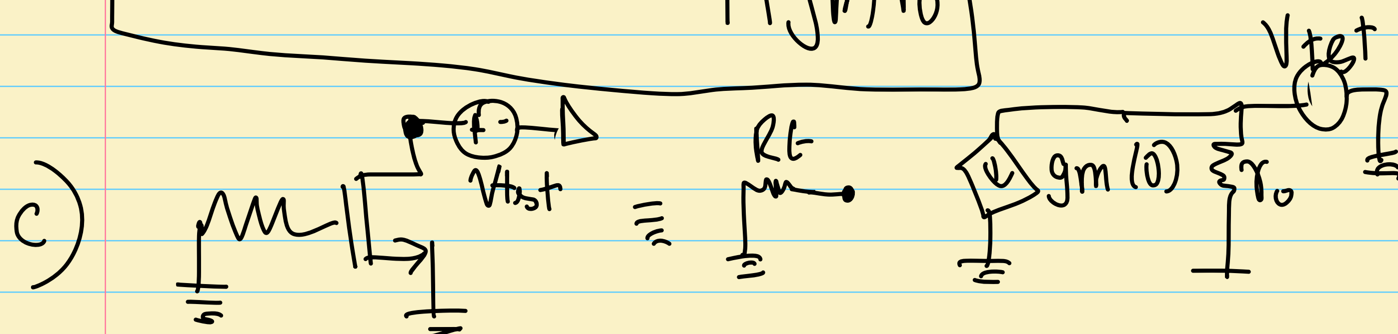
$$I_{test} = \frac{V_{test}\left[g_m R_S + R_S/r_o\right]}{R_S\left[1 + g_m R_S + R_S/r_o\right]}.$$

$$\therefore R_{eq} = \frac{R_S\left[1 + g_m R_S + R_S/r_o\right]}{g_m R_S + R_S/r_o}$$

$$= R_S + \frac{R_S}{g_m R_S + R_S/r_o}$$

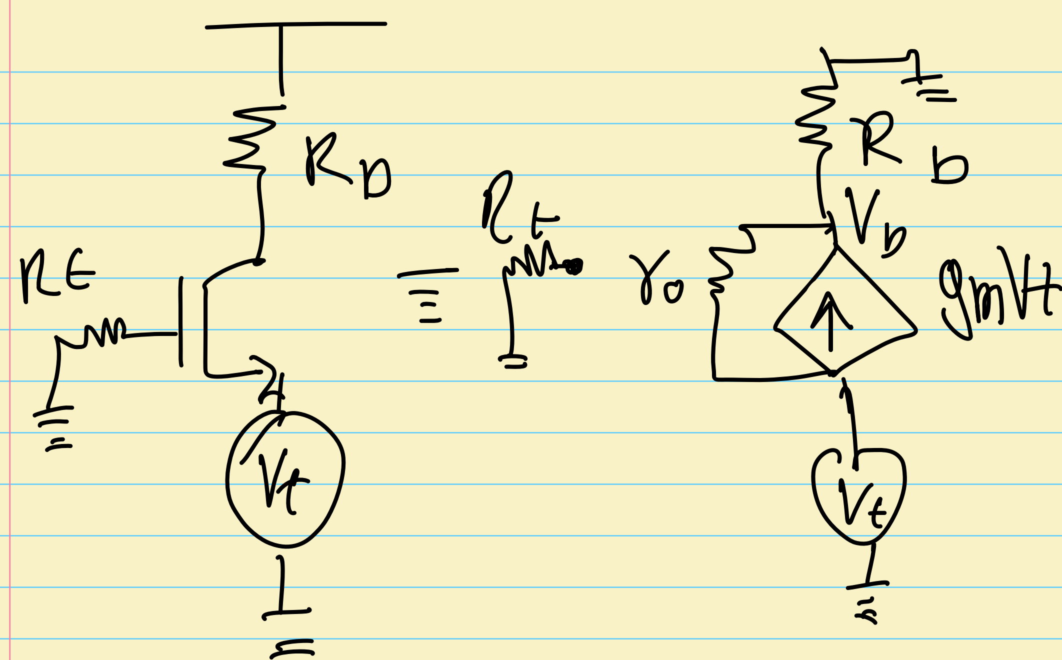
$$\boxed{R_{eq} = R_S + \frac{r_o}{1 + g_m r_o}}$$

c)



$$\boxed{r_{eq} = r_o}$$

d)



$$I_t = \frac{V_D}{R_D} = \frac{V_t - V_D}{r_o} + g_m V_t,$$

$$\therefore \frac{r_o V_D}{R_D} = V_t - V_D + g_m r_o V_t$$

$$\therefore V_D\left[1 + \frac{r_o}{R_D}\right] = V_t\left[1 + g_m r_o\right]$$

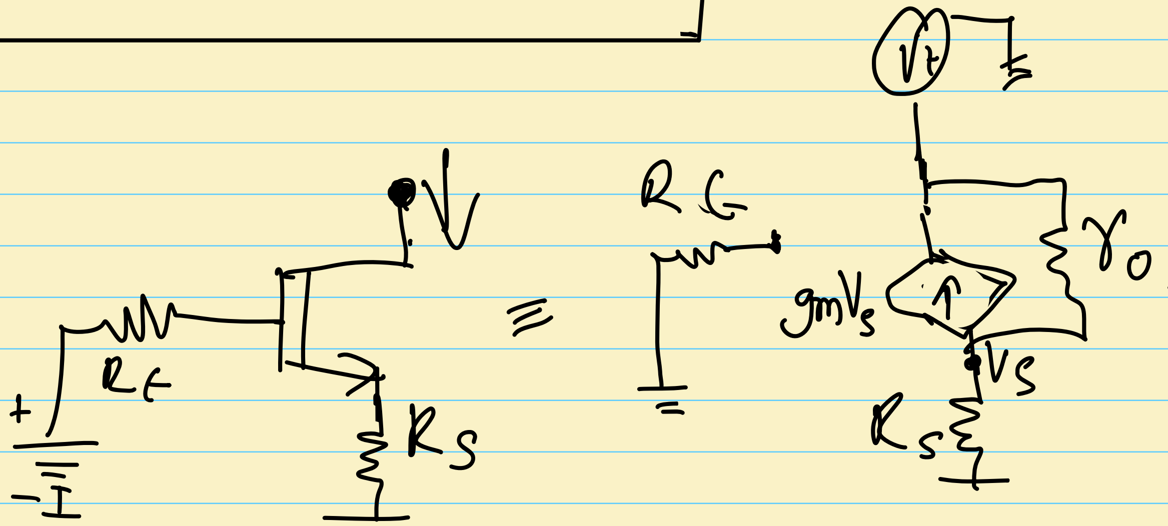
$$\therefore V_D = V_t \frac{\left[1 + g_m r_o\right]}{\left[1 + \frac{r_o}{R_b}\right]}$$

$$\therefore I_t = \frac{V_t\left[1 + g_m r_o\right]}{R_D\left[1 + r_o/R_b\right]}$$

$$\therefore I_t = \frac{V_t[1+g_m r_o]}{R_D + r_o}.$$

$$\therefore R_{eq} = \frac{R_D + r_o}{1 + g_m r_o}$$

c)



$$\therefore I_t = \frac{V_s}{R_S} = -g_m V_s + \frac{V_t - V_s}{r_o}$$

$$\therefore \frac{V_s r_o}{R_s} = -g_m r_o V_s + V_t - V_s.$$

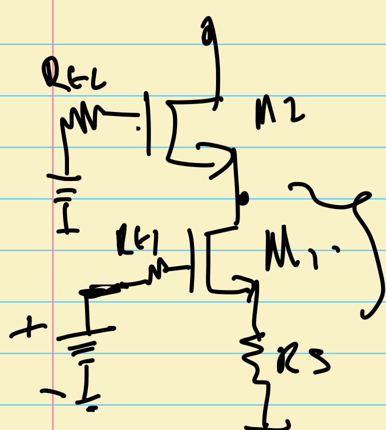
$$\therefore V_t = V_s\left[\frac{r_o}{R_S} + g_m r_o + 1\right]$$

$$\frac{V_t}{R_S\left[\frac{r_o}{R_S} + g_m r_o + 1\right]} = I_t.$$

$$\frac{V_t}{r_o + g_m r_o R_s + R_s} = I_t$$

$$\boxed{R_{eq} = R_s + r_o + g_m r_o R_s}$$

2.
a)



$R_{Seq} = R_s + r_{o_1} + g_{m_1} r_{o_1} R_s$.

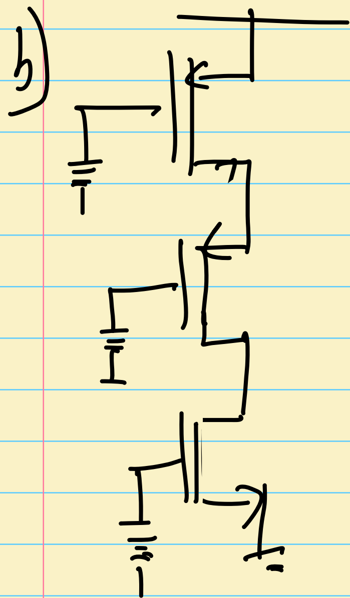
$$R_{eq} = R_{seq} + r_{o_2} + g_{m_2} r_{o_2} R_{seq}.$$

$$R_{eq} = \underbrace{R_s + r_{o_1} + g_{m_1} r_{o_1} R_s}_{R_{seq}} + r_{o_2}$$

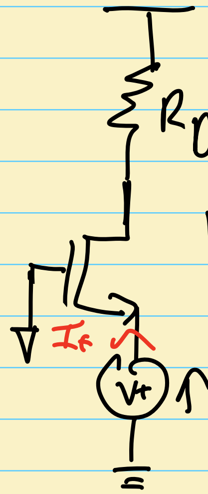
$$+ g_{m_2} r_{o_2} R_s + g_{m_2} r_{o_2} r_{o_1}$$

$$+ g_{m_2} g_{m_1} r_{o_2} r_{o_1} R_s.$$

b)



$\equiv$

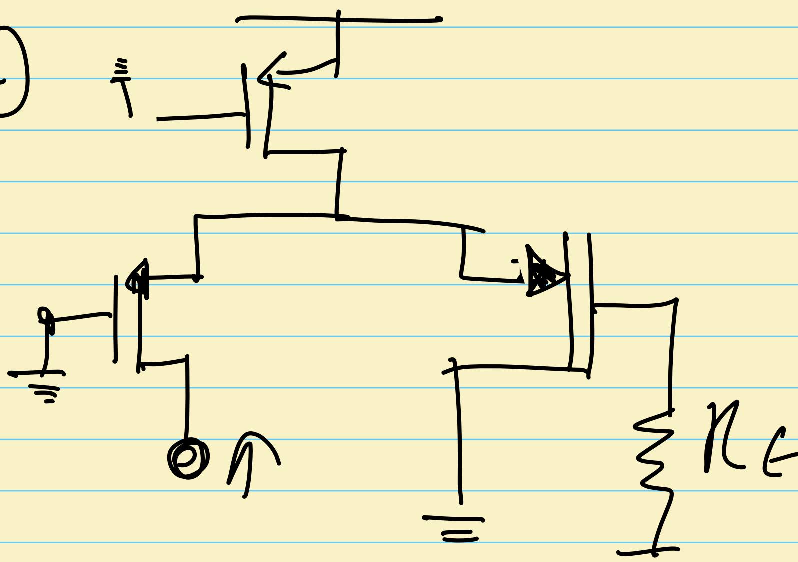


$$R_{Deq} = r_{o3} + r_{o2} + g_{m2} r_{o2} r_{o3}$$

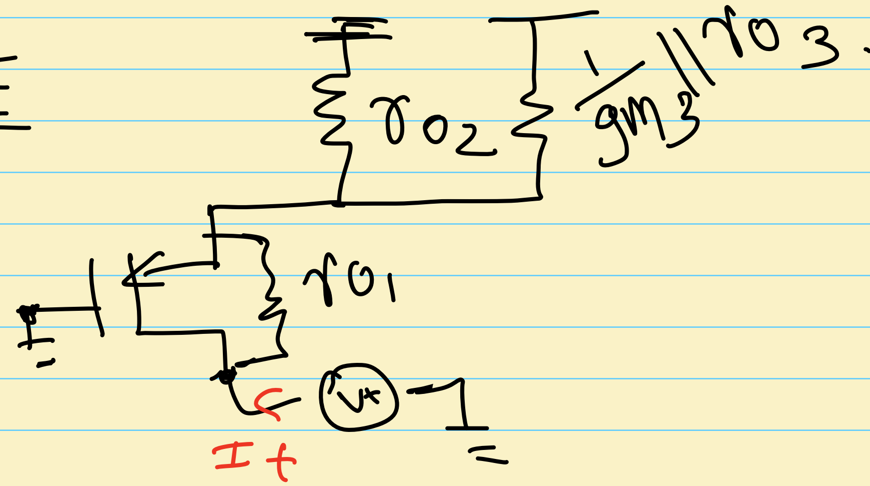
$$\boxed{R_{eq} = \frac{R_{Deq} + r_{o1}}{1 + g_{m1} r_{o1}}}$$

$$R_{eq} = \frac{r_{o3} + r_{o2} + g_{m2} r_{o2} r_{o3} + r_{o1}}{1 + g_{m1} r_{o1}}.$$

c)



$\equiv$

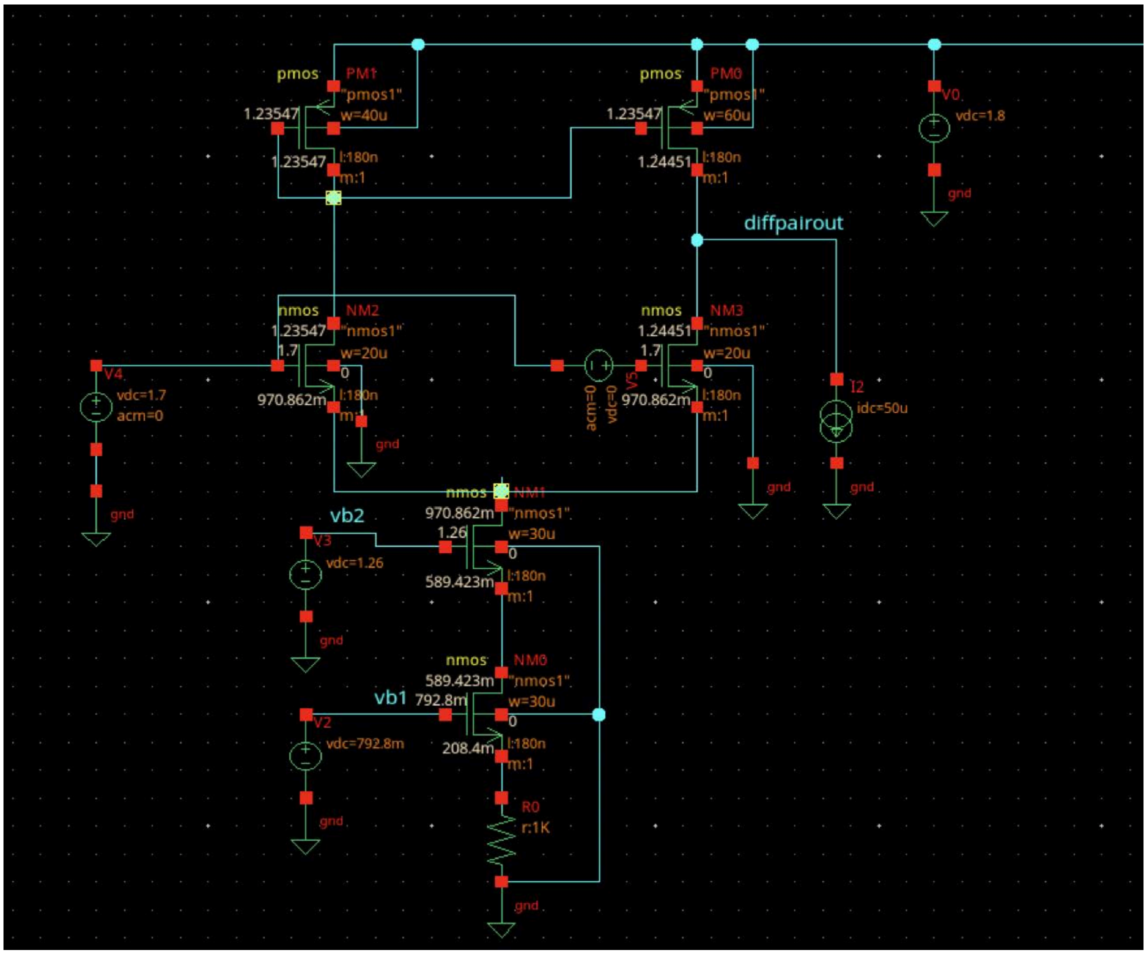


$$R_{eq} = r_{o1} + r_{o2} \| \frac{1}{g_{m3}} \| r_{o3} + g_{m1} \cdot r_{o1} \cdot \left(r_{o2} \| \frac{1}{g_{m3}} \| r_{o3} \right)$$

Q3.

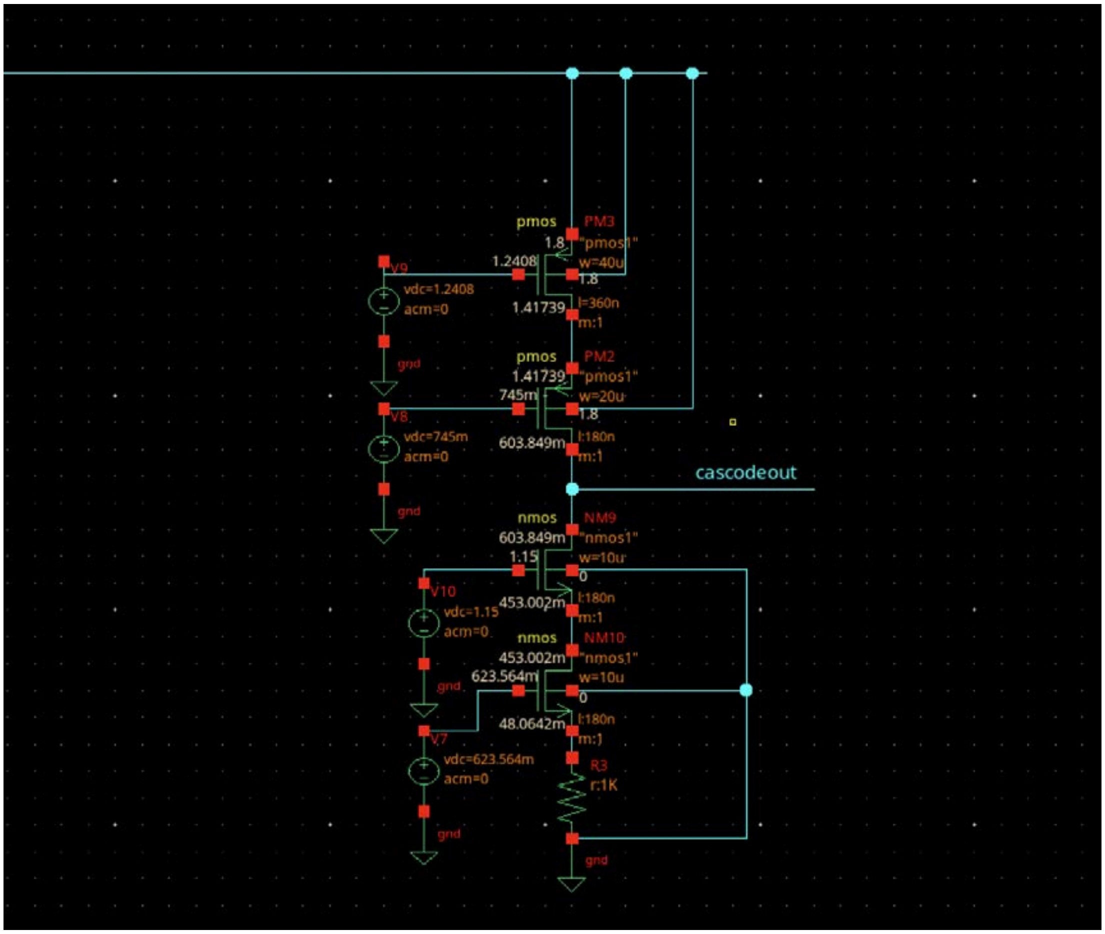
<u>Transistor bias voltages:</u>

Diff pair:



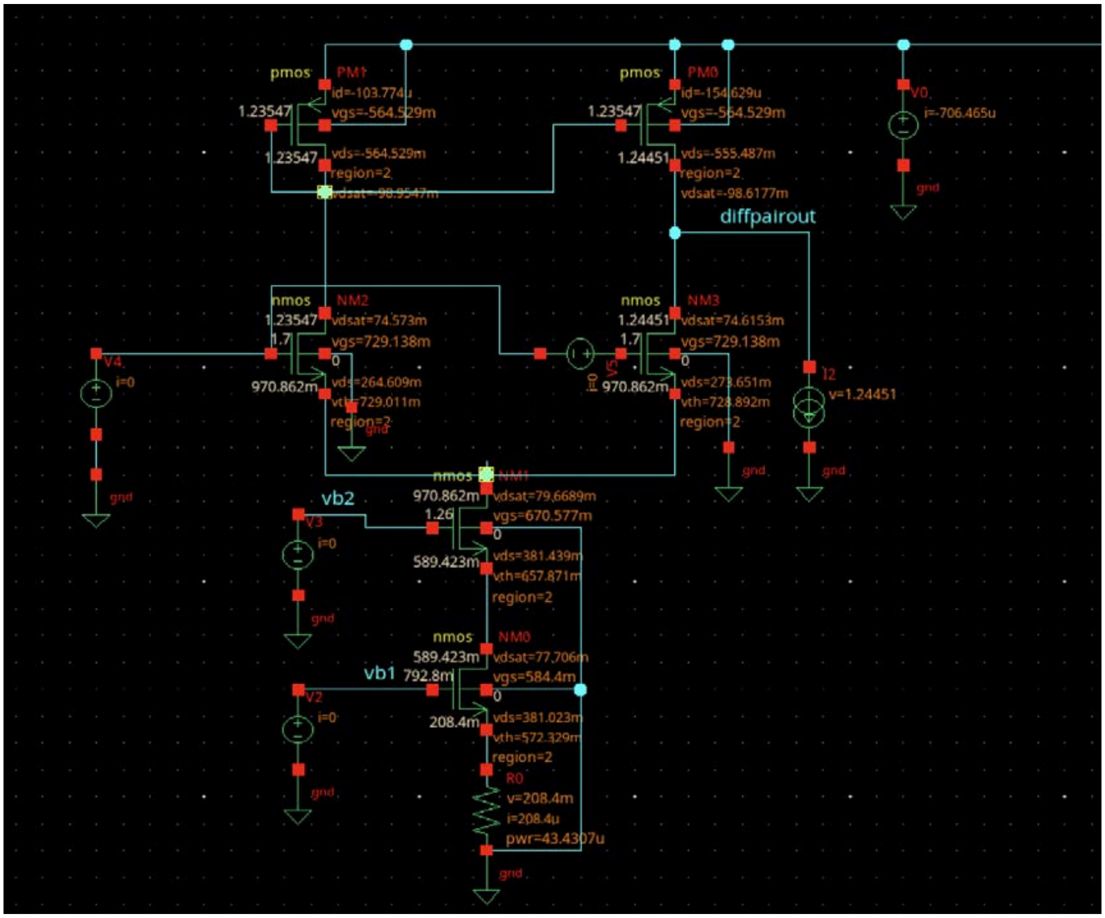
The gates of NM2 and NM3 are biased at 1.7V. Voltage source V5 is set to 0 V.

Cascode :

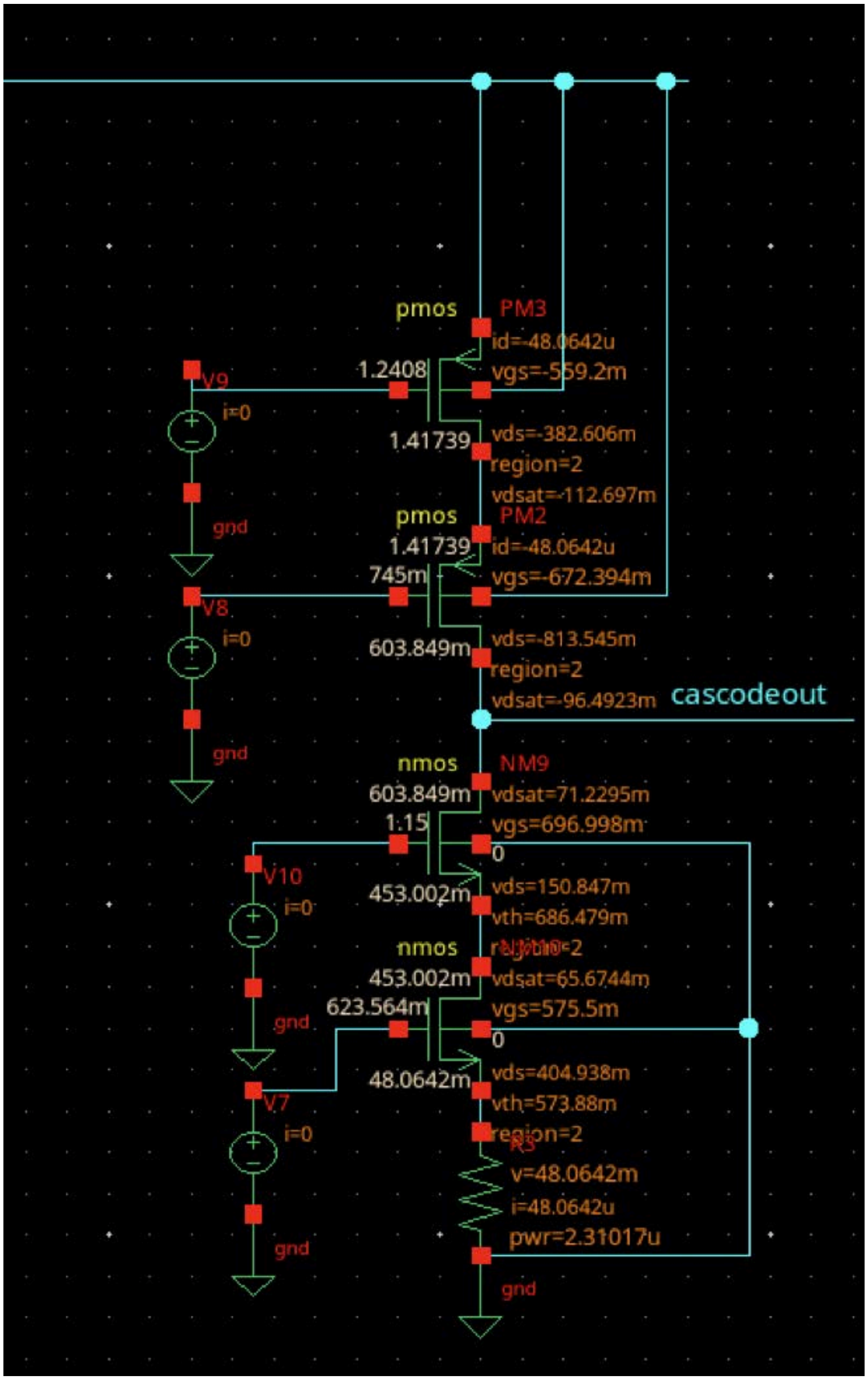


pmos
PM3
1.8
"pmos1"
1.2408
w=40u
1.8
1.41739
l=360n
m:1
V9
vdc=1.2408
acm=0
gnd
pmos
PM2
1.41739
"pmos1"
745m
w=20u
1.8
603.849m
l:180n
m:1
V8
vdc=745m
acm=0
gnd
cascodeout
nmos
NM9
603.849m
"nmos1"
1.15
w=10u
0
453.002m
l:180n
m:1
V10
vdc=1.15
acm=0
gnd
nmos
NM10
453.002m
"nmos1"
623.564m
w=10u
0
48.0642m
l:180n
m:1
V7
vdc=623.564m
acm=0
gnd
R3
r:1K
gnd

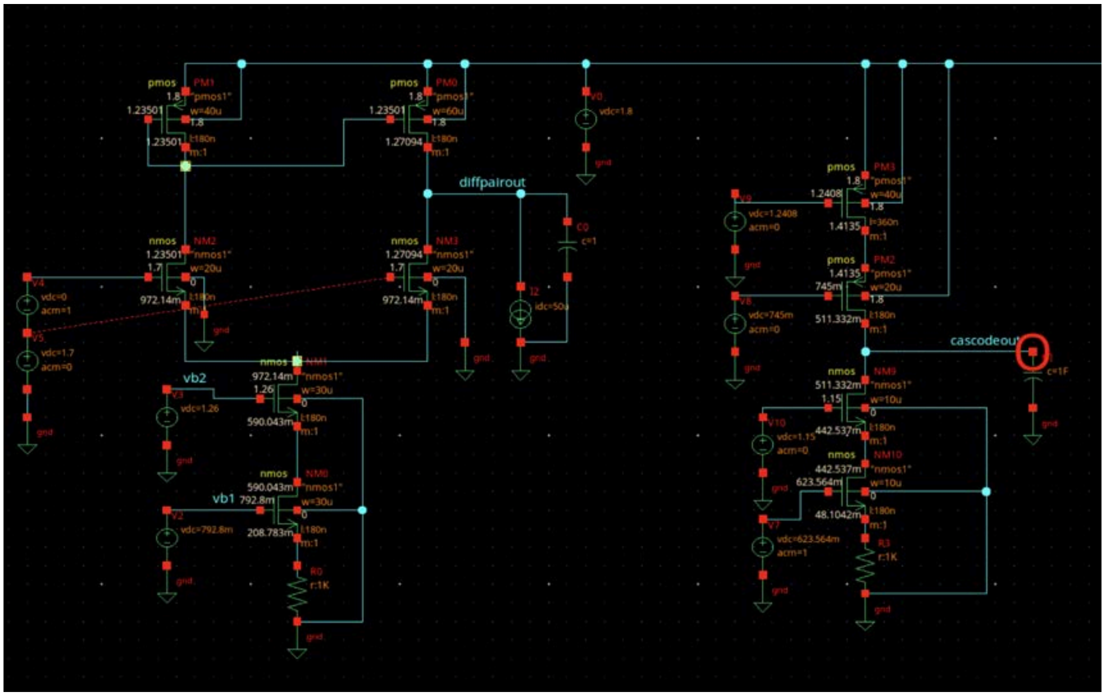
Regions of operation and DC operating points:
Diff pair:



Cascode DC operating points:

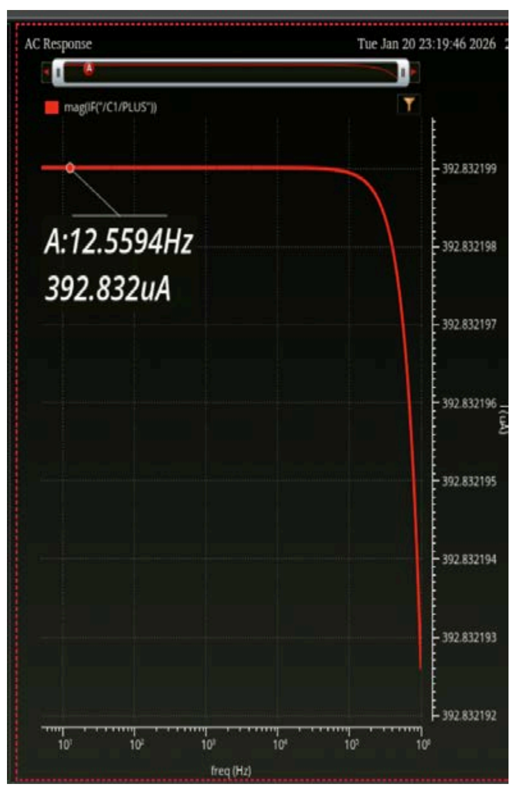


Gm calculation: I put a large capacitor at the output node, gave an AC input of 1 volt to the input transistor and measured the output current going into the large capacitor.

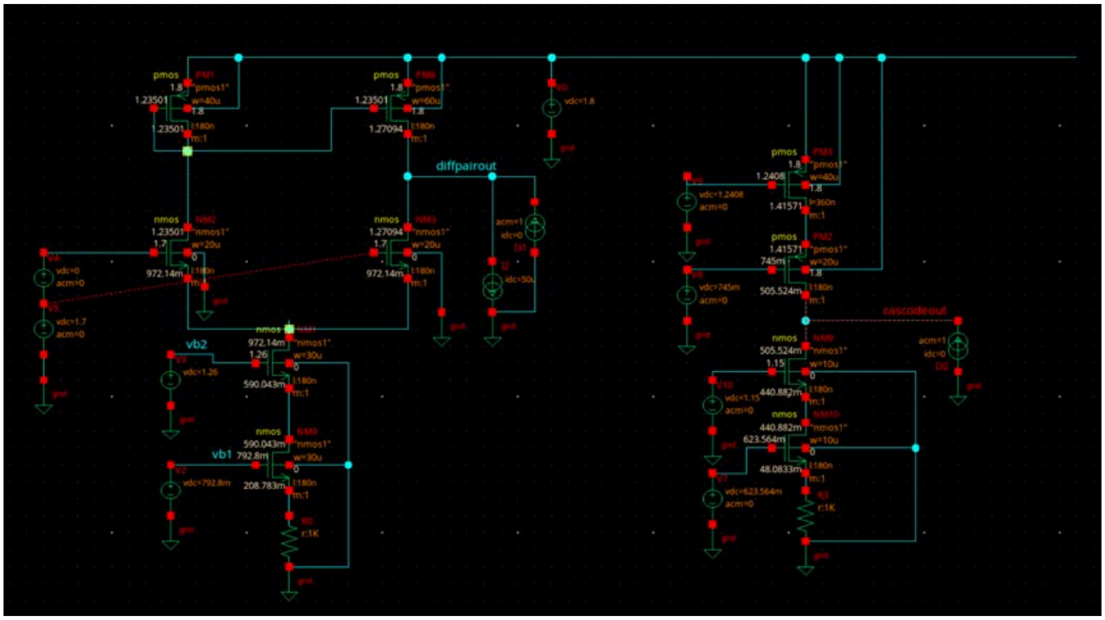


Diffpair measured Gm = 2.18521mS

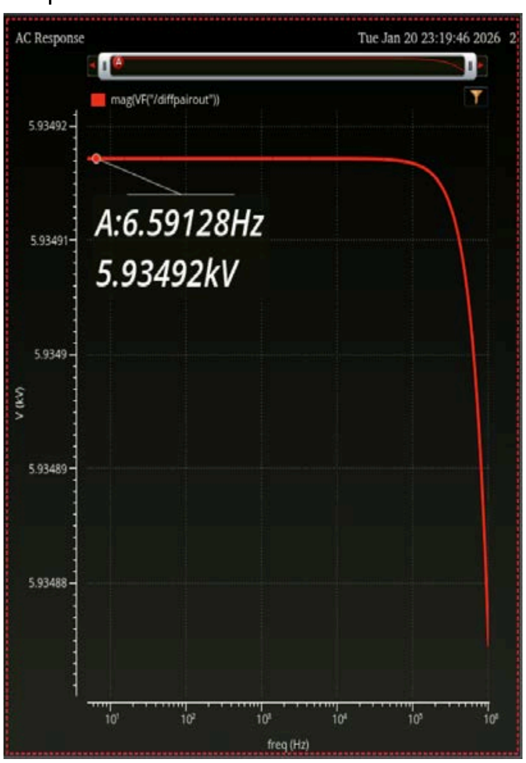
Cascode measured Gm = 392.832 uS



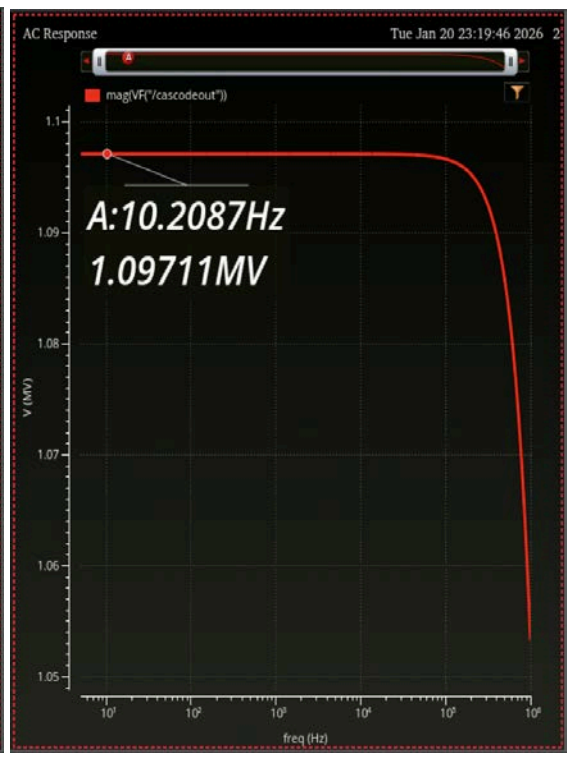
Rout calculation: I put an AC current source of 1 A at the output, shorted the AC inputs to gnd, then measured the voltage developed at the output node.



Diff pair Rout: 5.93492 Kilo Ohms

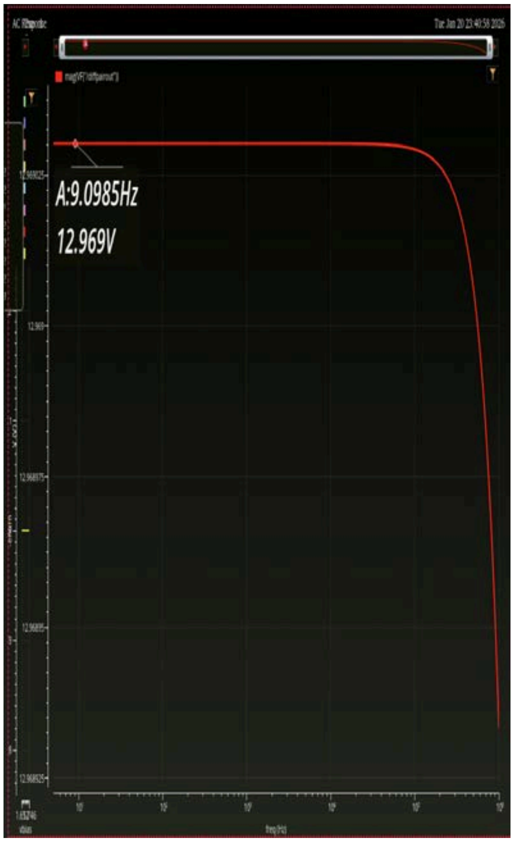


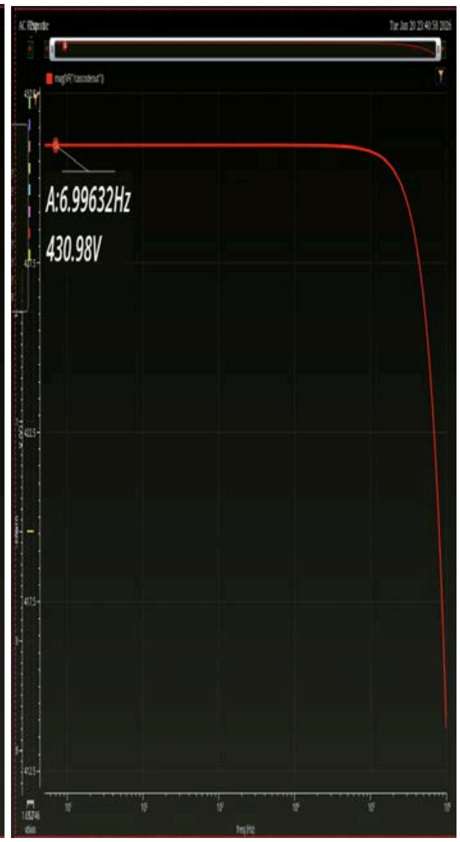
Cascode Rout: 1.09711 Mega Ohms



Diffpair Gm*Rout gain = 2.18521 mS * 5.93492 kohm = 12.96904653Cascode Gm*Rout gain = 392.832 uS * 1.09711 Mega Ohms = 430.9799155

Measured gain through regular AC gain simulation:





Diff pair gain: 12.969 V/V

Cascode gain = 430.98 V/V

The gains from both methods match