Problem 1

Grey and Meyer textbook derivation

The complete transfer function can be found by carrying out an analysis similar to that performed for Fig. 9.21, which gives

$$\frac{v_o}{i_s} = \frac{g_m R_1 R_2\left[1 - sC\left(\frac{1}{g_m} - R_Z\right)\right]}{1 + bs + cs^2 + ds^3} \tag{9.45}$$

where

$$b = R_2(C_2 + C) + R_1(C_1 + C) + R_Z C + g_m R_1 R_2 C \tag{9.46a}$$

$$c = R_1 R_2(C_1 C_2 + CC_1 + CC_2) + R_Z C(R_1 C_1 + R_2 C_2) \tag{9.46b}$$

$$d = R_1 R_2 R_Z C_1 C_2 C \tag{9.46c}$$

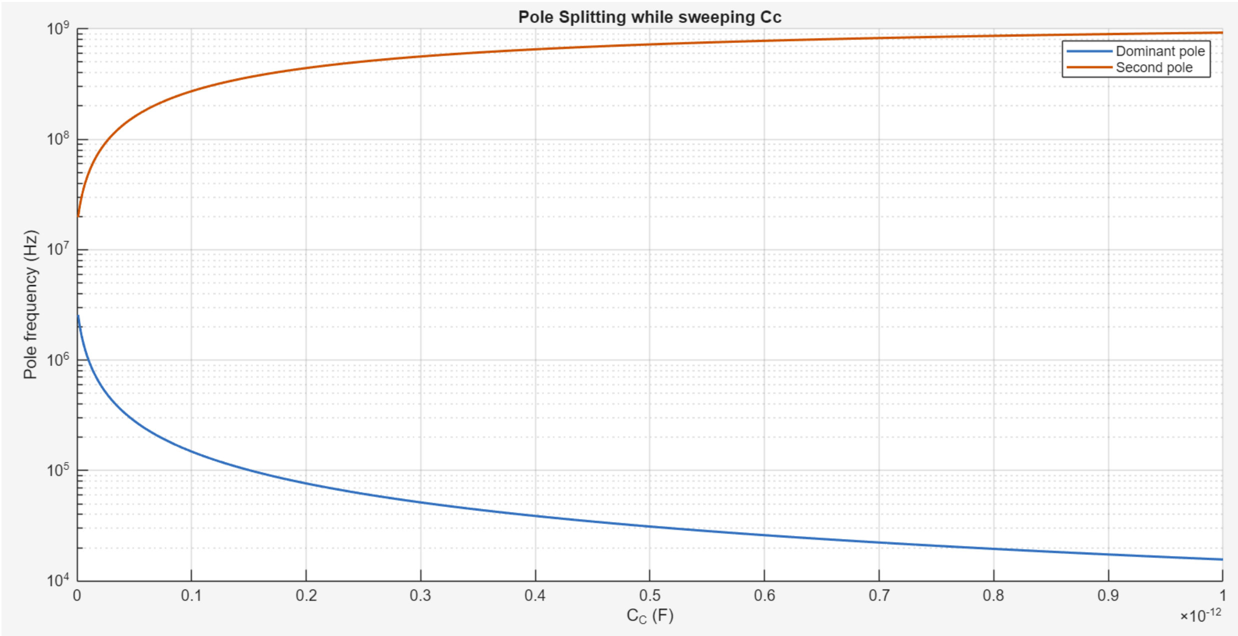
Again assuming $g_m R_1, g_m R_2 \gg 1$ and C is large, the poles can be approximated by

$$p_1 \approx -\frac{1}{g_m R_2 R_1 C} \tag{9.47a}$$

$$p_2 \approx -\frac{g_m C}{C_1 C_2 + C(C_1 + C_2)} \approx -\frac{g_m}{C_1 + C_2} \tag{9.47b}$$

$$p_3 \approx -\frac{1}{R_Z C_1} \tag{9.47c}$$

I included the approximations (p1, p2, p3) for reference but I will use the exact values b, c, d in this assignment. Also for reference C in the textbook refers to the coupling capacitor C_C as noted in class.



Problem 1 Cc sweep

I think a reasonable size for Cc is 0.25pF which places pole 1 at 61 kHz and pole 2 at 505 MHz.

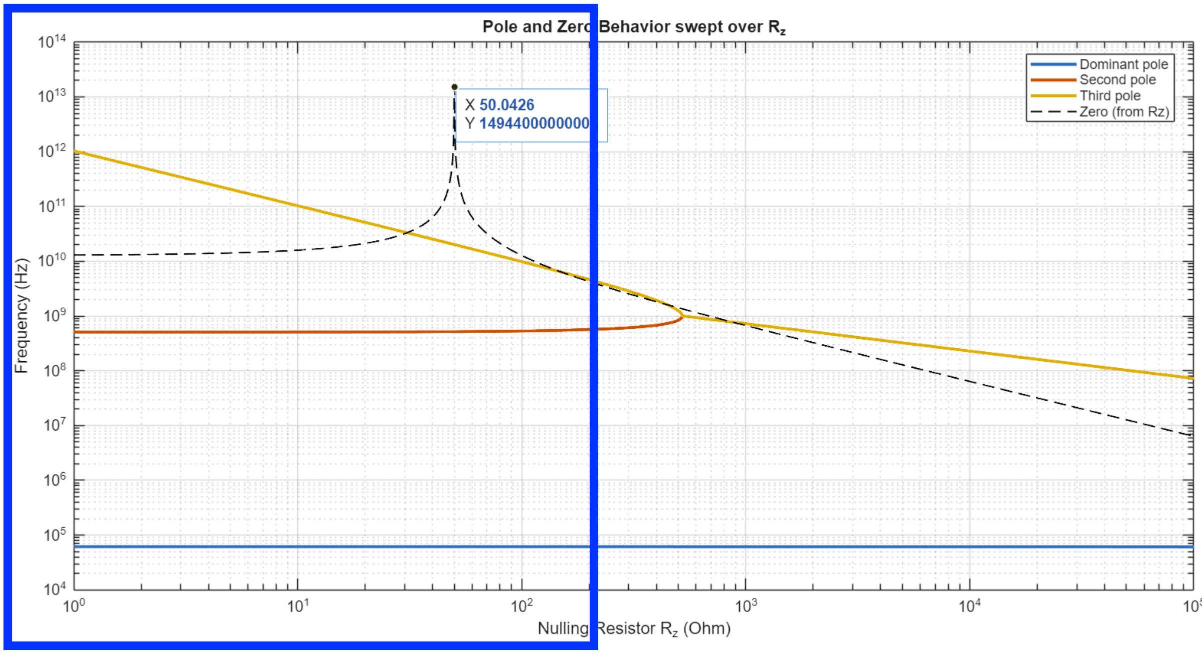
Now when introducing Rz, the system becomes a 3 pole system with a zero.

The zero of (9.45) is

$$z = \frac{1}{\left(\frac{1}{g_m} - R_Z\right)C} \tag{9.48}$$

This zero initially starts as a right half plane zero then transitions to a left half plane zero when Rz = 1/gm.

Outside of this box is meaningless, because one would never correct the RHP zero enough where after it becomes LHP zero (after 50 ohms) it starts to become lower frequency than 2nd and 3rd poles. It looks like after about 500 ohms the 2nd and 3rd poles meet. And perhaps they split into two complex poles? Or they just meet and stay together...? I didn't know anything about this because I never went there (one would never do that with RHP zero cancellation). One problem with plotting pole/zero magnitudes instead of showing them in s-plane (sigma and jw) is that I can't see what's really happening.



Problem 1 Rz sweep

From this plot we can see that the zero makes the transition at 50 ohms which equals 1/gm (1/0.02=50). This plot shows where an acceptable value of R_Z is. It has to be at least 50 ohms to be a zero near infinity, however it can not be so great that it causes the zero to come before the second pole.

1b.

The denominator of the equation does not change because there is symmetry between R1, R2 and C1, C2.

```
s^3 term: R1*R2*Rz*C1*C2*Cc;
```

Does not change when R1, R2 and C1, C2 swapped.

```
s^2 term = R1*R2*(C1*C2 + Cc*C1 + Cc*C2) + Rz*Cc*(R1*C1 + R2*C2);
```

Does not change when R1, R2 and C1, C2 swapped.

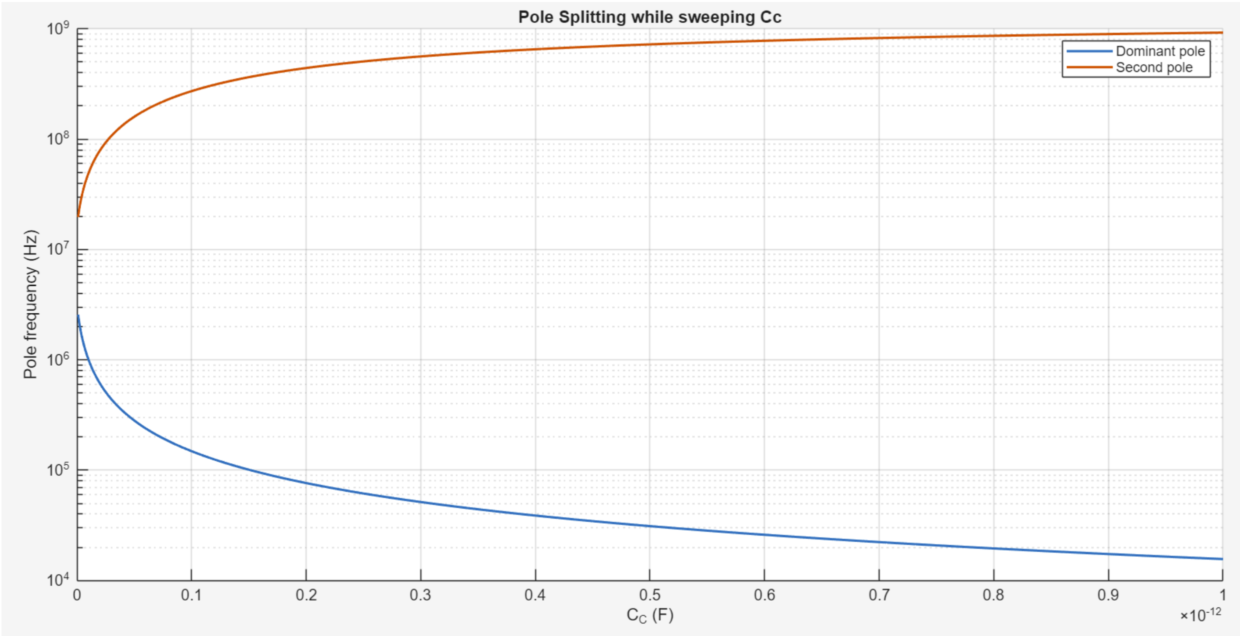
```
s^1 term = R2*(C2+Cc) + R1*(C1+Cc) + Rz*Cc + gm2*R1*R2*Cc;
```

Does not change when R1, R2 and C1, C2 swapped.

```
constant term = 1;
```

Does not change when R1, R2 and C1, C2 swapped.

Since none of the terms in the denominator change when R1, R2 and C1, C2 are swapped the pole locations are the same. Although the pole locations are the same, the circuit behavior can differ. I simulated the equation with swapped values and the got same result as problem 1, seen below.

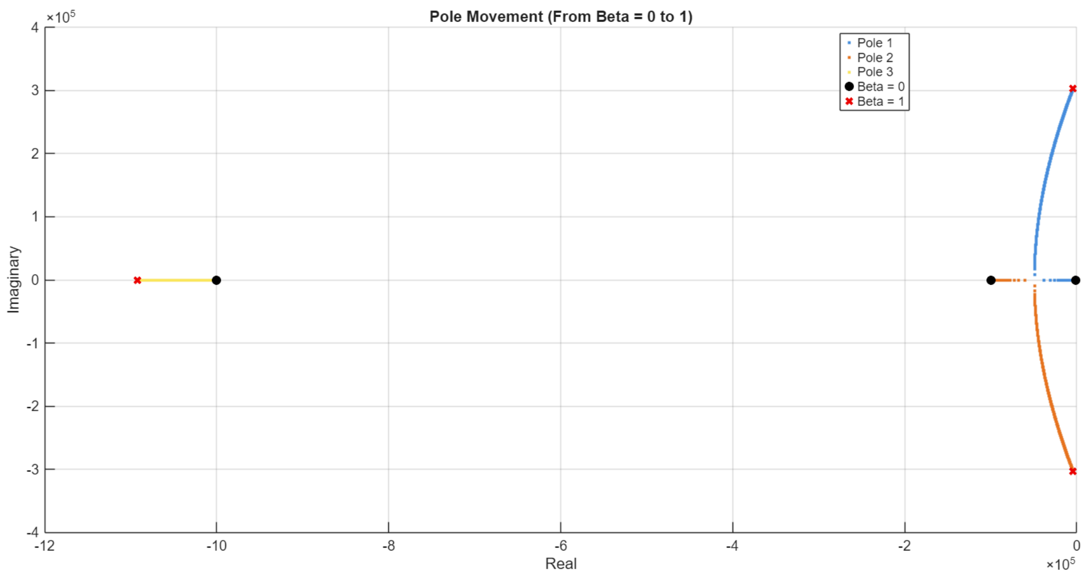


Problem 1b Cc sweep

2.

Gain = A(s) / (1 + B(A(s))

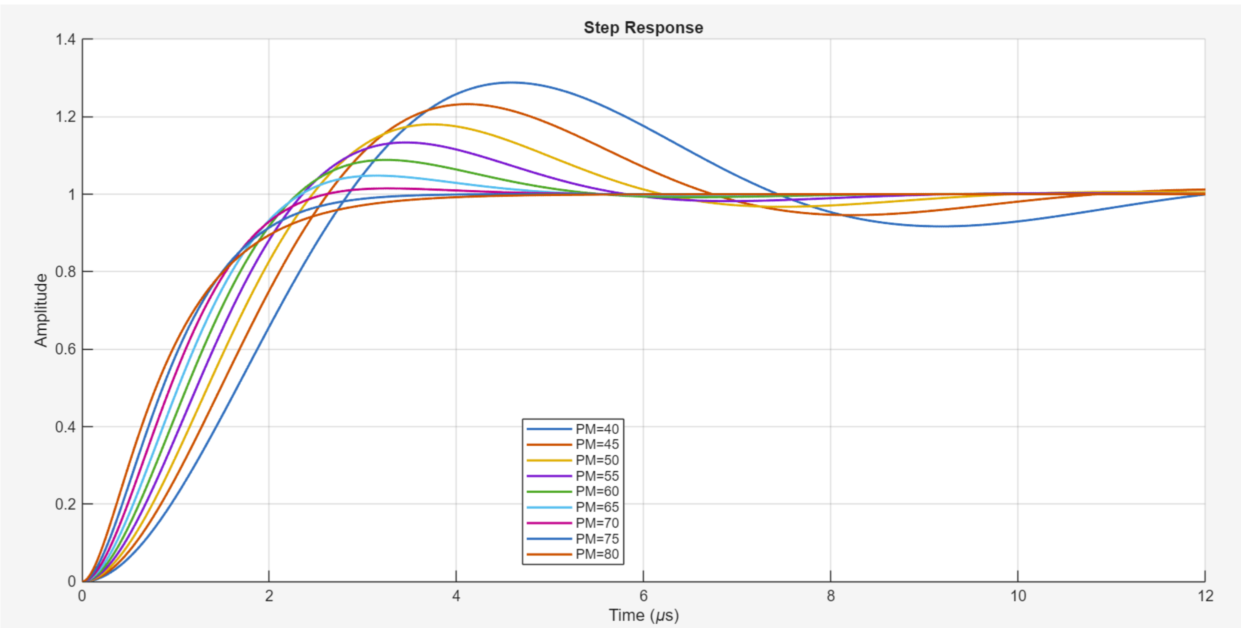
From this plot we can see that p1 and p2 collide and become complex conjugates. When beta equals one they are still in the left half plane which means they are stable but they are very close so the system might be underdamped. For reference, when the angle between the origin and each point is very small (more real than imaginary), the system will be overdamped.

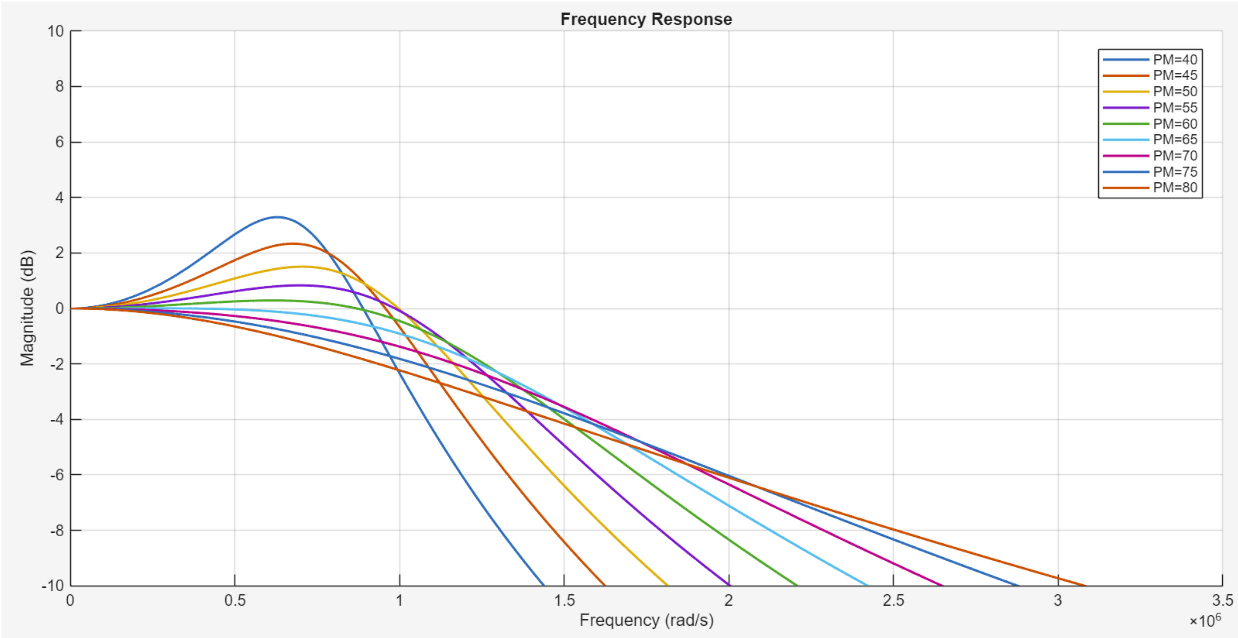


Problem 3 rootlocus plot (not using root locus cmd)

3.

Phase Margin	Overshoot % (time domain)	Peaking_dB (frequency domain)
40.098	28.817	3.29
45.082	23.233	2.3353
50.099	18.017	1.5058
54.917	13.347	0.83177
59.911	8.8504	0.28817
64.901	4.7908	0.0048372
69.91	1.4899	0
74.904	0.010482	0
79.905	0	0





4.

I initially thought the phase margin would greatly improve because the UGBW would happen sooner. But what I failed to realize was that the first pole was pushed out 4 decades and the DC gain was dropped 80 dB which essentially counteracts each other under the -20 dB/decade approximation. What I see in simulation remains mostly the same except for some slight differences because the math does not approximate. The same table is shown below. The lowest phase margin changes from 40.09 to 40.99 and causes the peaking to drop from 3.29 to 3.19 dB.

New_PM	Overshoot	Peaking
40.99	28.223	3.1929
45.893	22.784	2.2741
50.846	17.347	1.4702
55.618	13.187	0.81357
60.575	8.3883	0.28223
65.535	4.7042	0.0047347
70.521	1.412	0
75.498	0.012613	0
80.487	0	0

P2 locations (in MHz)								
PM=40	PM=45	PM=50	PM=55	PM=60	PM=65	PM=70	PM=75	PM=80
0.542	0.710	0.917	1.16	1.49	1.93	2.56	3.57	5.52