

Parameter Finding Methods for Oscillators with a Specified Oscillation Frequency

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ABSTRACT

This paper presents a generalized formulation of the periodic steady-state analysis for oscillators. The new formulation finds the value of a circuit parameter that results in a desired oscillation frequency for the circuit. Numerical methods based on the time-domain finite difference and shooting methods, and the frequency-domain harmonic balance method are described. Comparisons with search-based methods demonstrate the efficacy of the new approach.

Categories and Subject Descriptors

B.7.2 [Integrated Circuits]: Design Aids—*simulation*

General Terms

Algorithms

Keywords

Oscillators, periodic steady-state simulation, finite difference method, shooting method, harmonic balance method, oscillator design and optimization

1. INTRODUCTION

Oscillators are important blocks for clock generation in digital systems and for frequency up- and down-conversion in radio frequency communication systems. In all of these applications, the oscillator fundamental frequency f_0 is a design specification. During the design phase, the oscillator circuit is analyzed with periodic steady-state (PSS) simulations. However, the conventional PSS analysis [2], [3] treats the oscillation period $T = 1/f_0$, a given parameter, as an unknown. This is undesirable from a designer's perspective.

As an example, consider the problem of finding the control voltage for a voltage-controlled oscillator (VCO) that

yields the specified oscillation frequency. This problem cannot be solved in most existing analysis tools without a post-processing search-based approach. Several conventional PSS analyses for different control voltage values have to be carried out before the proper control voltage is determined. A more efficient and elegant solution to this problem is the PSS analysis for voltage and current controlled oscillators [1]. This method treats the control voltage or current as an unknown, while the oscillation frequency is a specified parameter.

In general, the control voltage of a VCO is not an appropriate frequency-tuning circuit parameter for use in the design phase. This is clear from the example of a phase-locked loop (PLL) where the VCO frequency is adjusted by the control voltage in a closed-loop configuration. Therefore, in designing an open-loop VCO, the control voltage cannot be used as the frequency-tuning parameter. Instead, the tank capacitor or the device size in a delay cell must be used as a frequency tuning parameter in LC or ring oscillators, respectively. Since the method described in [1] works specifically with a control voltage or current as the frequency tuning parameter, an alternative formulation is required.

In this paper, a generalized formulation of the PSS analysis for oscillators is presented. We call it PSS analysis with a specified oscillation frequency (PSS-SF). The new formulation is capable of working with a control voltage or current, as well as any circuit parameter that affects the oscillation frequency.

The concept of the PSS-SF analysis is introduced in Section 2. A general mathematical description for oscillators is provided. Based on this description the difference between the conventional and proposed PSS analyses in terms of the problem formulation and performance is explained. In Section 3, a discrete-time oscillator representation suitable for computer simulation [4] is presented. Based on this model the time-domain finite difference and shooting methods, as well as the frequency-domain harmonic balance method for PSS-SF analysis are presented. In Section 4, iterative search-based methods for finding the value of a circuit parameter that employ a conventional PSS analysis are presented. In Section 5, simulation results for LC and ring oscillator circuits are given. It is shown that the PSS-SF analysis is considerably faster than the search-based approaches. Finally, the paper is concluded in Section 6.

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2. THEORETICAL FORMULATION

In this section, the concept of oscillator PSS analysis with a specified oscillation frequency is reviewed. The PSS-SF formulation is compared to the conventional one, based on a general continuous-time mathematical representation for oscillators.

2.1 Continuous-Time Oscillator Equations

Any nonlinear oscillator circuit can be modeled as a set of m differential-algebraic equations (DAEs) given by

$$\dot{q}(x(t), \gamma_{f_0}) + f(x(t), \gamma_{f_0}) + b(\gamma_{f_0}) = 0 \quad (1)$$

where

$$\begin{aligned} t \in \mathbb{R} & : \text{time, independent variable,} \\ \gamma_{f_0} \in \mathbb{R} & : \text{oscillator circuit parameter,} \\ x : \mathbb{R} \rightarrow \mathbb{R}^m & : \text{oscillator state variables,} \\ q : \mathbb{R}^m \times \mathbb{R} \rightarrow \mathbb{R}^m & : \text{contribution of reactive components,} \\ f : \mathbb{R}^m \times \mathbb{R} \rightarrow \mathbb{R}^m & : \text{contribution of resistive components,} \\ b : \mathbb{R} \rightarrow \mathbb{R}^m & : \text{independent sources.} \end{aligned}$$

The T -periodic solution $x(t)$ of DAEs in (1) is called the PSS solution if it satisfies $x(t) = x(t + T)$. This periodicity constraint can be expressed as

$$x(0) = x(T) \quad (2)$$

Notice that if $x(t)$ is a PSS solution, then $x(t + \tau)$, $\forall \tau$ is also a valid PSS solution. A unique isolated solution can be selected by imposing a phase condition

$$\varphi(x(0)) = 0, \quad \varphi : \mathbb{R}^m \rightarrow \mathbb{R} \quad (3)$$

One possible phase condition is to let a component of $x(0)$ be a fixed value.

The oscillator PSS is uniquely defined by the system of (1), (2), and (3), resulting in the continuous-time equations for the oscillator in the steady-state

$$\begin{cases} \dot{q}(x(t), \gamma_{f_0}) + f(x(t), \gamma_{f_0}) + b(\gamma_{f_0}) = 0 \\ x(0) = x(T) \\ \varphi(x(0)) = 0 \end{cases} \quad (4)$$

This is a periodic boundary value problem (BVP), a special case of a two-point BVP [5].

2.2 Conventional PSS Analysis vs PSS-SF

A conventional PSS analysis computes the periodic waveform $x(t)$ and the oscillation period T , for a given parameter γ_{f_0}

$$\gamma_{f_0} \rightarrow \boxed{\text{Eq. (4)}} \rightarrow \{x(t), T\} \quad (5)$$

Here, the period T is one of the unknowns, and the circuit parameter γ_{f_0} is a parameter of the oscillator equations.

The idea behind the proposed PSS analysis is to swap the role of T for the role of γ_{f_0} , i.e., to introduce γ_{f_0} as an unknown, and treat the period T as a known parameter. The objective of the PSS-SF analysis is to find the value of the circuit parameter γ_{f_0} , and the periodic waveform $x(t)$ for a given oscillation period T

$$T \rightarrow \boxed{\text{Eq. (4)}} \rightarrow \{x(t), \gamma_{f_0}\} \quad (6)$$

According to (5), the oscillation period can be treated as a function $T(\gamma_{f_0})$, $T : \mathbb{R} \rightarrow \mathbb{R}$. The value of a circuit

parameter γ_{f_0} that results in an oscillation period T_{target} satisfies

$$F_T(\gamma_{f_0}) = T(\gamma_{f_0}) - T_{target} = 0 \quad (7)$$

The value of γ_{f_0} that satisfies (7) can be found iteratively by search-based methods with a conventional PSS analysis (5) at each iteration, or directly by the PSS-SF analysis (6).

In this paper, a single frequency-tuning parameter is used to satisfy a single design constraint, namely the oscillation frequency. Thus, the problem is well posed and has a unique solution. Future work will address multiple design parameters and objectives (oscillation frequency, signal amplitude, power consumption, etc.).

In contrast to [1], our formulation is general, as it works with a control voltage $b(\gamma_{f_0} \equiv V_{ctrl})$, or a control current $b(\gamma_{f_0} \equiv I_{ctrl})$, as well as any circuit parameter that affects the oscillation frequency. For example, a tank capacitor $q(x(t), \gamma_{f_0} \equiv C_{tank})$, MOSFET width $f(x(t), \gamma_{f_0} \equiv W)$, $q(x(t), \gamma_{f_0} \equiv W)$, etc.

3. NUMERICAL METHODS FOR PSS-SF ANALYSIS

In this section, numerical methods for computing the oscillator steady-state with a specified oscillation frequency based on a discrete-time oscillator description are presented.

3.1 Discrete-Time Oscillator Equations

Analysis of nonlinear oscillators using the continuous-time representation (4) is impractical. For numerical time-domain PSS analysis, time is discretized and the time-derivative operator is replaced by a finite-difference approximation. As an example, using uniformly spaced timepoints $t_i = ih$, $i \in \mathbb{N}$ and applying the backward Euler method, a simple discrete counterpart of (4) is

$$\begin{cases} \dot{q}_i + f_i + b = 0, \quad i = 1, \dots, n \\ x_0 = x_n \\ \varphi(x_0) = 0 \end{cases} \quad (8)$$

where

$$\begin{aligned} \dot{q}_i &= \frac{1}{h}(q_i - q_{i-1}), & x_i &\equiv x(t_i), \\ q_i &= q(x_i, \gamma_{f_0}), & t_i &= ih, \\ f_i &= f(x_i, \gamma_{f_0}), & h &= h(T) = T/n, \\ b &= b(\gamma_{f_0}). \end{aligned}$$

The discrete-time description in (8) is a system of $nm + m + 1$ nonlinear algebraic equations. The equations are written in terms of $(n + 1)m$ PSS waveform samples x_i , $i = 0, \dots, n$, the circuit parameter γ_{f_0} , and the oscillation period T . As proposed in Section 2.2 the circuit parameter γ_{f_0} and x_i are the unknowns, and the oscillation period T is a known parameter.

3.2 Finite Difference Method

The equations in (8) can be written in the following form

$$\begin{bmatrix} \frac{1}{h}(q_1 - q_n) + f_1 + b \\ \frac{1}{h}(q_2 - q_1) + f_2 + b \\ \vdots \\ \frac{1}{h}(q_n - q_{n-1}) + f_n + b \\ \hline \varphi(x_n) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \hline 0 \end{bmatrix} \quad (10)$$

$$J_{fd}(x_1, \dots, x_n, \gamma_{f_0}) = \begin{bmatrix} \frac{1}{h}C_1 + G_1 & & & & -\frac{1}{h}C_n & \frac{1}{h}\left(\frac{\partial q_1}{\partial \gamma_{f_0}} - \frac{\partial q_n}{\partial \gamma_{f_0}}\right) + \frac{\partial f_1}{\partial \gamma_{f_0}} + \frac{db}{d\gamma_{f_0}} \\ -\frac{1}{h}C_1 & \frac{1}{h}C_2 + G_2 & & & & \frac{1}{h}\left(\frac{\partial q_2}{\partial \gamma_{f_0}} - \frac{\partial q_1}{\partial \gamma_{f_0}}\right) + \frac{\partial f_2}{\partial \gamma_{f_0}} + \frac{db}{d\gamma_{f_0}} \\ & & \ddots & & & \vdots \\ & & & & -\frac{1}{h}C_{n-1} & \frac{1}{h}\left(\frac{\partial q_n}{\partial \gamma_{f_0}} - \frac{\partial q_{n-1}}{\partial \gamma_{f_0}}\right) + \frac{\partial f_n}{\partial \gamma_{f_0}} + \frac{db}{d\gamma_{f_0}} \\ \hline 0 & \dots & 0 & \frac{\partial \varphi(x_n)}{\partial x_n} & & 0 \end{bmatrix} \quad (9)$$

Note that the periodicity constraint $x_0 = x_n$ is not explicitly present in the above system. The periodicity constraint equations were used to eliminate x_0 from the list of unknowns. The remaining $nm + 1$ equations represent a finite difference formulation of the proposed PSS analysis. Denoting the left hand side of (10) by $F_{fd}(x_1, \dots, x_n, \gamma_{f_0})$, $F_{fd} : \underbrace{\mathbb{R}^m \times \dots \times \mathbb{R}^m}_n \times \mathbb{R}$ we rewrite the equations as

$$F_{fd}(x_1, \dots, x_n, \gamma_{f_0}) = 0 \quad (11)$$

The system of nonlinear equations in (11) can be solved using the Newton-Raphson iteration

$$J_{fd}(X_{fd}^{(k)}) [X_{fd}^{(k+1)} - X_{fd}^{(k)}] = -F_{fd}(X_{fd}^{(k)}) \quad (12)$$

where k is the iteration index, $X_{fd} = \begin{bmatrix} x_1^T & \dots & x_n^T \\ \vdots & & \vdots \\ \gamma_{f_0} \end{bmatrix}^T$ is the vector of the finite difference unknowns,

$$J_{fd}(x_1, \dots, x_n, \gamma_{f_0}) = \partial F_{fd} / \partial X_{fd} \quad (13)$$

is the augmented finite difference Jacobian matrix, given by (9), $J_{fd} : \underbrace{\mathbb{R}^m \times \dots \times \mathbb{R}^m}_n \times \mathbb{R} \rightarrow \mathbb{R}^{(nm+1) \times (nm+1)}$. The

Jacobian matrix is defined in terms of C_i and G_i , the capacitance and conductance matrices

$$C_i = \frac{\partial q_i}{\partial x_i} = \frac{\partial q(x, \gamma_{f_0})}{\partial x} \Big|_{x_i}, \quad C_i : \mathbb{R}^m \times \mathbb{R} \rightarrow \mathbb{R}^{m \times m} \quad (14)$$

$$G_i = \frac{\partial f_i}{\partial x_i} = \frac{\partial f(x, \gamma_{f_0})}{\partial x} \Big|_{x_i}, \quad G_i : \mathbb{R}^m \times \mathbb{R} \rightarrow \mathbb{R}^{m \times m} \quad (15)$$

For large problems, fast preconditioned iterative methods [1], [6] are employed to solve the linear system in (12).

The last column of the Jacobian matrix in (9) requires $\partial q / \partial \gamma_{f_0}$, $\partial f / \partial \gamma_{f_0}$, and $db / d\gamma_{f_0}$. These derivatives with respect to the frequency-tuning parameter can be obtained analytically or numerically from device models.

The Newton-Raphson method is a method with local convergence, therefore, the initial guess must be close enough to the solution. In particular, the parameter value $\gamma_{f_0}^{(0)}$ must be such that the circuit oscillates. Even with a good initial guess and existence of a solution, it is possible that the circuit stops being an oscillator in the middle of the Newton-Raphson iterative loop. Such a situation is untypical for the conventional PSS analysis, and it requires special treatment to recover, such as roll-back and damping.

There may be no solution to the problem in (7), which means that the circuit can not oscillate at the desired frequency, independent of the parameter value. In this case, at

some iteration of the PSS-SF Newton loop, the frequency-tuning parameter is updated to a value for which the circuit stops oscillating. The convergence behavior is then similar to that of a conventional PSS analysis for a circuit that does not have a PSS solution. The method will either converge to the trivial DC solution, or will not converge.

3.3 Shooting Method

The last $m + 1$ equations in (8) represent a shooting formulation of the PSS-SF analysis

$$\begin{bmatrix} x_n - x_0 \\ \vdots \\ \varphi(x_0) \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} \quad (16)$$

where x_0 and γ_{f_0} are the unknowns. Given γ_{f_0} , x_n is obtained from a transient analysis with the initial condition x_0 using the first nm equations in (8). After the solution x_0 and γ_{f_0} are found, the remaining PSS waveform samples x_i , $i = 1, \dots, n$ are obtained from an additional transient analysis.

Let us rewrite the shooting method formulation in (16) as

$$F_{sh}(x_0, \gamma_{f_0}) = 0 \quad (17)$$

This system of nonlinear equations can be solved using the Newton-Raphson iteration

$$J_{sh}(X_{sh}^{(k)}) [X_{sh}^{(k+1)} - X_{sh}^{(k)}] = -F_{sh}(X_{sh}^{(k)}) \quad (18)$$

where k is the iteration index, $X_{sh} = \begin{bmatrix} x_0^T \\ \vdots \\ \gamma_{f_0} \end{bmatrix}^T$ is the vector of the shooting method unknowns,

$$J_{sh}(x_0, \gamma_{f_0}) = \partial F_{sh} / \partial X_{sh} \quad (19)$$

is the Jacobian, $J_{sh} : \mathbb{R}^m \times \mathbb{R} \rightarrow \mathbb{R}^{(m+1) \times (m+1)}$, given by

$$J_{sh}(x_0, \gamma_{f_0}) = \begin{bmatrix} \frac{\partial x_n}{\partial x_0} - I & \frac{\partial x_n}{\partial \gamma_{f_0}} \\ \vdots & \vdots \\ \frac{\partial \varphi(x_0)}{\partial x_0} & 0 \end{bmatrix} \quad (20)$$

where I is the identity matrix.

Computation of the Jacobian requires differentiation of the first nm equations in (8) with respect to x_0 and γ_{f_0}

$$\left[\frac{1}{h}C_i + G_i \right] \frac{\partial x_i}{\partial x_0} = \frac{1}{h}C_{i-1} \frac{\partial x_{i-1}}{\partial x_0} \quad (21)$$

$$\begin{aligned} \left[\frac{1}{h}C_i + G_i \right] \frac{\partial x_i}{\partial \gamma_{f_0}} &= \frac{1}{h}C_{i-1} \frac{\partial x_{i-1}}{\partial \gamma_{f_0}} \\ &\quad - \frac{1}{h} \left(\frac{\partial q_i}{\partial \gamma_{f_0}} - \frac{\partial q_{i-1}}{\partial \gamma_{f_0}} \right) - \frac{\partial f_i}{\partial \gamma_{f_0}} - \frac{db}{d\gamma_{f_0}} \end{aligned} \quad (22)$$

The derivatives $\partial x_n / \partial x_0$ and $\partial x_n / \partial \gamma_{f_0}$ are obtained from (21) and (22) iteratively for $i = 1, \dots, n$ starting from the initial conditions $\partial x_0 / \partial x_0 = I$ and $\partial x_0 / \partial \gamma_{f_0} = 0$.

$$J_{hb}(X_{hb}) = \left[\begin{array}{c|c} j\Omega \Gamma \begin{bmatrix} C_0 \\ \vdots \\ C_{n-1} \end{bmatrix} \Gamma^{-1} + \Gamma \begin{bmatrix} G_0 \\ \vdots \\ G_{n-1} \end{bmatrix} \Gamma^{-1} & j\Omega \Gamma \begin{bmatrix} \frac{\partial q_0}{\partial \gamma_{f_0}} \\ \vdots \\ \frac{\partial q_{n-1}}{\partial \gamma_{f_0}} \end{bmatrix} + \Gamma \begin{bmatrix} \frac{\partial f_0}{\partial \gamma_{f_0}} \\ \vdots \\ \frac{\partial f_{n-1}}{\partial \gamma_{f_0}} \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ \frac{\partial b}{\partial \gamma_{f_0}} \\ 0 \end{bmatrix} \\ \hline \dots \quad \frac{\partial \varphi_{hb}}{\partial X_{-2}} \quad \frac{\partial \varphi_{hb}}{\partial X_{-1}} \quad \frac{\partial \varphi_{hb}}{\partial X_0} \quad \frac{\partial \varphi_{hb}}{\partial X_1} \quad \frac{\partial \varphi_{hb}}{\partial X_2} \quad \dots & 0 \end{array} \right] \quad (23)$$

3.4 Harmonic Balance Method

The n -periodic discrete-time waveforms x_i can be uniquely represented as an n -periodic sequence of impulses in the frequency domain at multiples of the oscillation frequency f_0 . Given f_0 , the harmonic balance analysis for oscillators with a specified frequency finds the parameter γ_{f_0} and n Fourier coefficients X_k , $X_k \in \mathbb{C}^m$, $k = \dots, -1, 0, +1, \dots$ of the PSS solution waveform

$$\begin{bmatrix} \vdots \\ X_{-1} \\ X_0 \\ X_1 \\ \vdots \end{bmatrix} = \Gamma \begin{bmatrix} x_0 \\ \vdots \\ x_{n-1} \end{bmatrix} \quad (24)$$

where $\Gamma : \underbrace{\mathbb{R}^m \times \dots \times \mathbb{R}^m}_n \rightarrow \underbrace{\mathbb{C}^m \times \dots \times \mathbb{C}^m}_n$ represents the discrete-time Fourier transform operator, defined by

$$X_k = \frac{1}{n} \sum_{i=0}^{n-1} x_i e^{-j2\pi ki/n} \quad (25)$$

After the Fourier coefficients X_k are found, the inverse Fourier transform Γ^{-1} is used to get the time-domain PSS solution

$$x_i = \sum_{k=0}^{n-1} X_k e^{j2\pi ki/n} \quad (26)$$

where n -periodicity of X_k was used.

The harmonic balance problem can be formulated as a system of $nm + 1$ nonlinear equations

$$\left[\begin{array}{c|c} j\Omega \Gamma \begin{bmatrix} q_0 \\ \vdots \\ q_{n-1} \end{bmatrix} + \Gamma \begin{bmatrix} f_0 + b \\ \vdots \\ f_{n-1} + b \end{bmatrix} \\ \hline \varphi_{hb}(\dots, X_{-1}, X_0, X_1, \dots) \end{array} \right] = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \quad (27)$$

where the unknowns are γ_{f_0} and X_k , $k = \dots, -1, 0, +1, \dots$. The first nm equations in (27) correspond to the first nm equations in (8). Notice that the equations in (27) are algebraic. The time-domain differentiation in (8) is replaced by a frequency domain multiplication with $j\Omega$,

$$j\Omega : \underbrace{\mathbb{C}^m \times \dots \times \mathbb{C}^m}_n \rightarrow \underbrace{\mathbb{C}^m \times \dots \times \mathbb{C}^m}_n$$

such that

$$\begin{bmatrix} \dot{q}_0 \\ \vdots \\ \dot{q}_{n-1} \end{bmatrix} = \Gamma^{-1} j\Omega \Gamma \begin{bmatrix} q_0 \\ \vdots \\ q_{n-1} \end{bmatrix} \quad (28)$$

with

$$\Omega = 2\pi f_0 \text{diag}([\dots, -I, 0, +I, \dots]) \quad (29)$$

where I is the identity matrix, $I \in \mathbb{R}^{m \times m}$, and $0 \in \mathbb{R}^{m \times m}$.

The periodicity constraint of (8) $x_n = x_0$ is not explicitly present in (27). It is enforced by the periodic nature of the complex exponential basis functions of the inverse Fourier transform in (26).

Similar to the last equation of (8), the last equation in (27), $\varphi_{hb} : \underbrace{\mathbb{C}^m \times \dots \times \mathbb{C}^m}_n \rightarrow \mathbb{R}$, is used to select a unique isolated

solution among an infinite set of valid phase-shifted solutions. A commonly used phase condition is to let the imaginary part of the first Fourier coefficient of a component of the PSS solution be zero.

Let us rewrite the harmonic balance equations in (27) as

$$F_{hb}(\dots, X_{-1}, X_0, X_1, \dots, \gamma_{f_0}) = 0 \quad (30)$$

This system of nonlinear equations can be solved using the Newton-Raphson iteration

$$J_{hb}(X_{hb}^{(k)}) [X_{hb}^{(k+1)} - X_{hb}^{(k)}] = -F_{hb}(X_{hb}^{(k)}) \quad (31)$$

where $X_{hb} = [\dots X_{-1} X_0 X_1 \dots \vdots \gamma_{f_0}]^T$ is the vector of the harmonic balance method unknowns, k is the iteration index,

$$J_{hb}(\dots, X_{-1}, X_0, X_1, \dots, \gamma_{f_0}) = \partial F_{hb} / \partial X_{hb} \quad (32)$$

is the augmented Jacobian matrix of the harmonic balance method, $J_{hb} : \underbrace{\mathbb{C}^m \times \dots \times \mathbb{C}^m}_n \times \mathbb{R} \rightarrow \mathbb{C}^{(nm+1) \times (nm+1)}$.

The Jacobian J_{hb} requires the sensitivities of the Fourier coefficients of q_i and f_i with respect to X_k . These sensitivities can be calculated analytically for linear circuit components and for devices which are defined in the frequency domain, such as delays and transmission lines. The sensitivities of the nonlinear resistive device contributions can be computed as

$$\frac{\partial \Gamma \begin{bmatrix} f_0 \\ \vdots \\ f_{n-1} \end{bmatrix}}{\partial \begin{bmatrix} \vdots \\ X_{-1} \\ X_0 \\ X_1 \\ \vdots \end{bmatrix}} = \Gamma \frac{\partial \begin{bmatrix} f_0 \\ \vdots \\ f_{n-1} \end{bmatrix}}{\partial \Gamma \begin{bmatrix} x_0 \\ \vdots \\ x_{n-1} \end{bmatrix}} = \Gamma \begin{bmatrix} G_0 & & \\ & \ddots & \\ & & G_{n-1} \end{bmatrix} \Gamma^{-1} \quad (33)$$

and require time domain evaluations of the conductance matrices G_i . The sensitivities due to the nonlinear reactive devices require the capacitance matrices C_i and can be computed in a manner similar to (33). The harmonic balance Jacobian matrix J_{hb} is given by (23).

4. SEARCH-BASED METHODS

The value of a parameter γ_{f_0} satisfying (7) can be found iteratively by root-finding methods. In these approaches the solution is obtained from a number of conventional PSS analyses

$$\begin{aligned} \gamma_{f_0}^{(0)} &\rightarrow \boxed{\text{Eq. (4)}} \rightarrow \{x(t)^{(0)}, T^{(0)}\} \\ &\vdots \\ \gamma_{f_0}^{(N)} &\rightarrow \boxed{\text{Eq. (4)}} \rightarrow \{x(t)^{(N)}, T^{(N)} = T_{target}\} \end{aligned}$$

4.1 Bisection Method

The bisection method searches for the root of (7) in the interval $[\gamma_{f_0L}^{(0)}, \gamma_{f_0H}^{(0)}]$, known to contain the root. At each iteration k the interval is halved $\gamma_{f_0M}^{(k)} = (\gamma_{f_0L}^{(k)} + \gamma_{f_0H}^{(k)})/2$, and the sub-interval containing the root is chosen for the next search

$$[\gamma_{f_0L}^{(k+1)}, \gamma_{f_0H}^{(k+1)}] = \begin{cases} [\gamma_{f_0L}^{(k)}, \gamma_{f_0M}^{(k)}], & F_T(\gamma_{f_0L}^{(k)})F_T(\gamma_{f_0M}^{(k)}) < 0 \\ [\gamma_{f_0M}^{(k)}, \gamma_{f_0H}^{(k)}], & \text{otherwise} \end{cases} \quad (34)$$

The value of the parameter $\gamma_{f_0}^{(k)}$ is chosen to be $\gamma_{f_0M}^{(k)}$.

The bisection method is a brute-force approach to finding the value of γ_{f_0} . This method requires only a conventional PSS analysis to evaluate F_T , therefore, existing simulators can be used as is. It is shown in Section 5 that the bisection method is computationally the most inefficient method.

4.2 Newton-Raphson Method

The Newton-Raphson iteration

$$\gamma_{f_0}^{(k+1)} = \gamma_{f_0}^{(k)} - \frac{F_T(\gamma_{f_0}^{(k)})}{\left. \frac{\partial F_T(\gamma_{f_0})}{\partial \gamma_{f_0}} \right|_{\gamma_{f_0}^{(k)}}} \quad (35)$$

requires computation of the sensitivity

$$\frac{\partial F_T}{\partial \gamma_{f_0}} = \frac{\partial T}{\partial \gamma_{f_0}} \approx -\frac{e_{out}^T \frac{\partial x(T)}{\partial \gamma_{f_0}}}{e_{out}^T \dot{x}(T)} \approx -\frac{e_{out}^T \frac{\partial x_n}{\partial \gamma_{f_0}}}{e_{out}^T \frac{x_n - x_{n-1}}{h}} \quad (36)$$

where $\partial x_n / \partial \gamma_{f_0}$ is computed similar to the last column of the shooting method Jacobian matrix J_{sh} in (20) using (22) at the steady state. e_{out}^T is a unity vector that selects the output signal of an oscillator.

Notice that the Newton-Raphson method requires not only the function F_T but also the sensitivity $\partial T / \partial \gamma_{f_0}$. Thus, simulation tools must be capable of evaluating $\partial q / \partial \gamma_{f_0}$, $\partial f / \partial \gamma_{f_0}$, and $db / d\gamma_{f_0}$ to compute $\partial T / \partial \gamma_{f_0}$ analytically. The PSS solution at iteration k can be used as an initial guess at iteration $k+1$ to improve PSS convergence. It is shown in Section 5 that the Newton-Raphson method is faster than the bisection method, and slower than the new PSS-SF analysis.

5. EXAMPLES AND RESULTS

We have implemented the PSS-SF analysis in our Matlab-based circuit simulator, and Berkeley Design Automation's RF FastSPICE. In this section, the problem (7) of finding a circuit parameter γ_{f_0} given the period T is solved for various LC and ring oscillators.

Consider a three stage ring oscillator in Figure 1. The

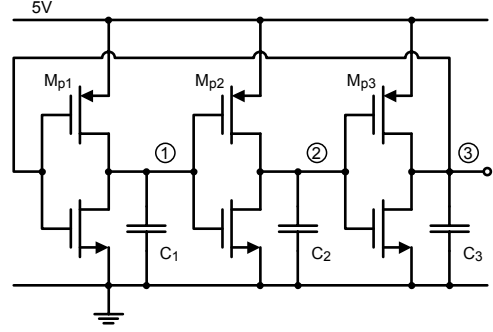


Figure 1: Schematic of a three-stage ring oscillator with inverter delay cell.

oscillation frequency is set by the gain and delay of the inverter stage. One way to change the oscillation frequency is to alter the MOSFET sizes. Let the width of the p -channel devices be $\gamma_{f_0} \equiv W_{Mp1} = W_{Mp2} = W_{Mp3}$ such that the oscillation period T_{target} is 3ns, and (7) is satisfied.

The problem in (7) was solved iteratively, by a sequence of conventional shooting PSS analyses

$$\begin{aligned} \gamma_{f_0}^{(0)} &\rightarrow \boxed{\text{PSS}} \boxed{N_C^{(0)}} \rightarrow \{x(t)^{(0)}, T^{(0)}\} \\ &\vdots \\ \gamma_{f_0}^{(N)} &\rightarrow \boxed{\text{PSS}} \boxed{N_C^{(N)}} \rightarrow \{x(t)^{(N)}, T^{(N)} \approx T_{target}\} \end{aligned}$$

using bisection and Newton-Raphson methods. $N_C^{(k)}$ is the number of iterations of PSS analysis at the k^{th} search iteration. The solution for γ_{f_0} is $13.84315\mu\text{m}$. Figure 2 shows the output voltage waveform $e_{out}^T x_i^{(k)}(k_C)$ at every iteration $k_C = 0, \dots, N_C$ of the PSS analysis, performed at every iteration $k = 0, \dots, N$ of the Newton-Raphson search method.

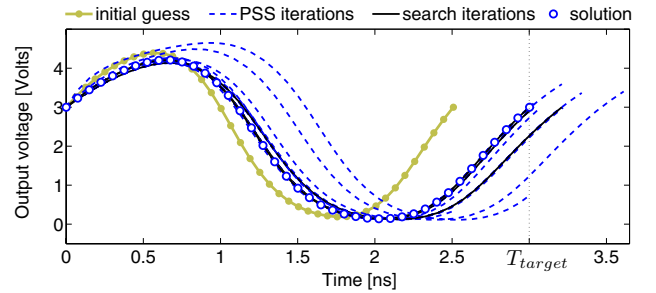


Figure 2: Convergence process of the Newton-Raphson search method.

The problem in (7) was also solved directly, using the shooting method for PSS-SF analysis starting from an initial circuit state $T(19.46331\mu\text{m}) = 2.508201\text{ns}$

$$T_{target} \rightarrow \boxed{\text{PSS-SF}} \boxed{N_{SF}} \rightarrow \{x_i, \gamma_{f_0}\}$$

where N_{SF} is the number of iterations of PSS-SF analysis. The PSS-SF solution for γ_{f_0} is $13.84359\mu\text{m}$, and it agrees with the solution found by the Newton-Raphson method. Figure 3 shows the output voltage waveform $e_{out}^T x_i^{(k_{SF})}$ at every PSS-SF iteration $k_{SF} = 0, \dots, N_{SF}$.

Oscillator	Cross-coupled LC -tank		Three-stage ring (inverter)		Colpitts		Three-stage ring (Maneatis)		
Parameter	$\gamma_{f_0} \equiv L$	$\gamma_{f_0} \equiv C$	$\gamma_{f_0} \equiv W_p$	$\gamma_{f_0} \equiv C$	$\gamma_{f_0} \equiv L$	$\gamma_{f_0} \equiv C$	$\gamma_{f_0} \equiv W_n$	$\gamma_{f_0} \equiv I_{bias}$	
Initial State	γ_{f_0}	25.330296nH	1.000000pF	19.46331 μ m	10.00000fF	12.66500 μ H	25.00000pF	3.000000 μ m	167.5000 μ A
	T	1.020443ns		2.508201ns		100.3709ns		1.208336ns	
Target	T	1.200000ns		3.000000ns		106.0000ns		1.300000ns	
PSS-SF Analysis	γ_{f_0}	35.016342nH	1.392773pF	13.84359 μ m	11.96076fF	14.13769 μ H	28.67896pF	1.330166 μ m	152.9561 μ A
	T	1.200000ns	1.200000ns	3.000000ns	3.000000ns	106.0000ns	106.0000ns	1.300000ns	1.300000ns
	N_{SF}	8	8	7	6	7	7	7	5
	N_{SF}/N_{SF}	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Bisection Method	γ_{f_0}	34.98813nH	1.391630pF	13.85133 μ m	11.95505fF	14.13792 μ H	28.67954pF	1.330931 μ m	152.9539 μ A
	T	1.199601ns	1.199516ns	2.999106ns	2.998567ns	106.0009ns	106.0008ns	1.299914ns	1.300018ns
	$\Sigma N_C^{(k)}$	2+10+8+8 \times 4	2+9+8+4 \times 4+4 \times 3	11+2+6+9 \times 5	2+9+6+8 \times 2	2+10+8 \times 4	2+6+8 \times 4	6+2+3 \times 4+8 \times 3	5+2+8 \times 3
	$\Sigma N_C^{(k)}/N_{SF}$	6.50	5.88	9.14	5.50	6.29	5.71	6.29	6.20
Newton Raphson	γ_{f_0}	35.011026nH	1.392789pF	13.84315 μ m	11.96072fF	14.13769 μ H	28.67898pF	1.329161 μ m	152.9236 μ A
	T	1.1999944ns	1.200005ns	3.000051ns	2.999983ns	106.0000ns	106.0000ns	1.300119ns	1.300220ns
	$\Sigma N_C^{(k)}$	2+8+5+4+2	2+7+5+4+2	2+8+6+5+4	2+7+4+4+2	2+7+4+2+2	2+8+4+2+2	2+4+4+5+4+3	2+6+4+3
	$\Sigma N_C^{(k)}/N_{SF}$	2.63	2.50	3.57	3.17	2.43	2.57	3.14	3.00

Table 1: Simulation results for several oscillator circuits.

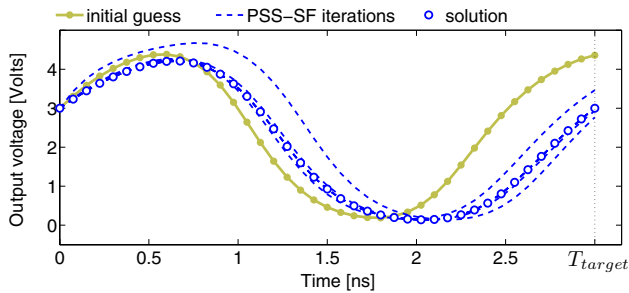


Figure 3: Convergence process of the shooting method for PSS-SF analysis.

Similar simulations were performed for $\gamma_{f_0} \equiv C_1 = C_2 = C_3$, and for various frequency-tuning parameters of an NMOS cross-coupled LC -tank oscillator, Colpitts oscillator, and a three-stage ring oscillator with the Maneatis delay cell [7].

The speed of the three approaches is compared based on the number of iterations N_{SF} of the PSS-SF analysis and the sum $\Sigma N_C^{(k)}$ of the conventional PSS iterations $N_C^{(k)}$ at every iteration $k = 0, \dots, N$ of a search-based method.

The simulation results are summarized in Table 1. For a given relative tolerance of $\epsilon_{rel} = 10^{-3}$, the PSS-SF analysis is about 3 times faster than the iterative Newton-Raphson method, and 6 times faster than the bisection method. The relative errors of γ_{f_0} and T for the three sets of results are within the simulation tolerance.

6. CONCLUSION

We have presented a general formulation and numerical methods for oscillator PSS-SF analysis, a PSS analysis with a specified oscillation frequency. The PSS-SF analysis finds the value of a circuit parameter that results in the circuit oscillating at the desired frequency. This makes the method well suited for most applications wherein the oscillator fun-

damental frequency is a design specification. The new formulation is general and handles any frequency-tuning circuit parameter. Furthermore, our new formulation is more efficient than search-based approaches that employ a conventional PSS analysis. Simulation results show that the PSS-SF analysis is in good agreement with the conventional PSS analysis.

7. ACKNOWLEDGMENT

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