

CAREER: Computing low cost maps between surfaces and curves

Amir Nayyeri, Oregon State University

The meta problem of finding a cost efficient (partial) map or deformation between two (embedded) surfaces or curves is not only interesting from a theoretical perspective, but also encapsulate several variants of the well-known shape matching and shape correspondence problems.

Surprisingly though, very little is known about different instances of this problem, particularly about maps between surfaces. In the abundance of applications, and because of this lack of understanding, heuristics with no mathematical guarantee have been applied, which are prone to failure as the input changes.

This proposal studies a wide range of problems under the umbrella of cost efficient maps and deformations between surfaces and curves. The goal of this study is to design exact or approximation algorithms and to understand limits of algorithms in this context. We mainly consider metric distortion and Fréchet length as cost functions for maps, two measures with several established applications. For deformations, we consider their height as the cost function. Viewing the deformation of a curve as a sequence of curves, that we call snapshots, its height is defined to be the snapshot with maximum length. In case of surfaces, the height is the snapshot with maximum area.

Intellectual merit. The research of this proposal will mainly contribute to the advancement of the algorithmic understanding of maps between surfaces and curves. A full understanding can only be obtained through an interplay among different areas such as geometry, topology, algorithmic graph theory and functional analysis. The PI seeks to combine ideas from these diverse theoretical and computational areas to tackle the above described questions, which are of interest for all mentioned fields. Moreover, being in the intersection of different research interests, this proposal provides a unique opportunity to create new connections among different communities.

Broader impact. Computing low cost maps between surfaces is a significant problem that many applications face. Surprisingly though, most of the current methods are heuristics with no provable guarantees. This is partly because exact or approximation algorithms for this problem will, most likely, involve sophisticated mathematical tools and ideas. This proposal will benefit several areas through establishing a deeper understanding of maps between surfaces, providing tools to develop more efficient algorithms for many applications. Naturally, given the wide spectrum of research areas touched by the topic of this proposal, it is an opportunity to initiate collaborations.

Integrated to all research efforts is the process of familiarizing students with the recent advancements in the field. To this end, the PI will develop two courses in algorithmic graph theory and computational geometry and topology to directly involve students in the most recent problems of the fields. Furthermore, the PI will be involved in educating and training undergraduate and graduate students. The undergraduate projects will be simpler problems related to this proposal with possibly light components of programming. This proposal will support two graduate students working closely together and taking part in advising the undergraduate students.