Homework 1

Do the following problems. Make sure to show your explanation. The odd-numbered problems have the answer key in the back of the book.

Problem 2 of 1.1: A certain state has driver's license numbers that consist of 1 letter followed by 6 digits. How many different driver's license numbers are possible in that state?

Problem 3 of 1.1: Suppose you own 6 jackets, 4 pairs of pants, 3 sweaters and 10 shirts. How many different outfits do you have?

Problem 5 of 1.1: Suppose that your schedule for next semester must consist of one natural science class, one liberal arts class, one humanities class, and one physical science class. How many ways can you make up your schedule if you can choose from 3 natural science classes, 4 liberal arts classes, 6 humanities classes and 3 physical science classes?

Problem 11 of 1.1: A telephone number consists of 10 digits: 3 digit area code followed by 7 digit number. How many different telephone numbers are possible if neither the first digit of the area code nor the first digit of the number itself can be 0?

Problem 4 of 1.2: Compute and compare the following values:

(a) $3! - 1!$ and $(3-1)!$

(b) $4! + 2!$ and $(4+2)!$

(c) $\frac{4!}{2!}$ and $\left(\frac{4}{2}\right)!$

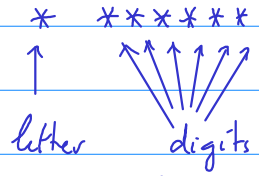
(d) $3! \cdot 2!$ and $(3 \cdot 2)!$

Problem 9 of 1.2: Find the smallest number n for which $n!$ is larger than the population of the world.

Additional problem: A passcode consists of 3 characters chosen from 3 categories: the alphabet, the digits, and the 9 symbols $\sim$, !, @, #, \$, %, ^, &, *. How many different passcodes can be created if there must be one character from each category?

Problem 2 of 1.1:

- Task: make a license plate of the form

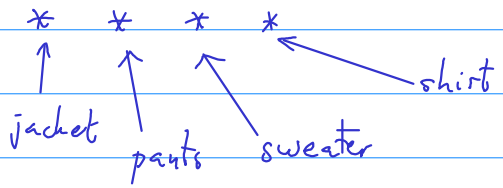


Step 1: choose a letter. There are 26 ways.
Step 2: choose a digit for the first slot. There are 10 ways.
Step 3: choose a digit for the second slot. There are 10 ways.
Step 4: " third "
Step 5: " fourth "
Step 6: " fifth "
Step 7: " sixth "

Therefore, there are $26 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 26,000,000$ ways to make a license plate. That is the number of license plates possible.

Problem 3 of 1.1

- Task: make an outfit of the form

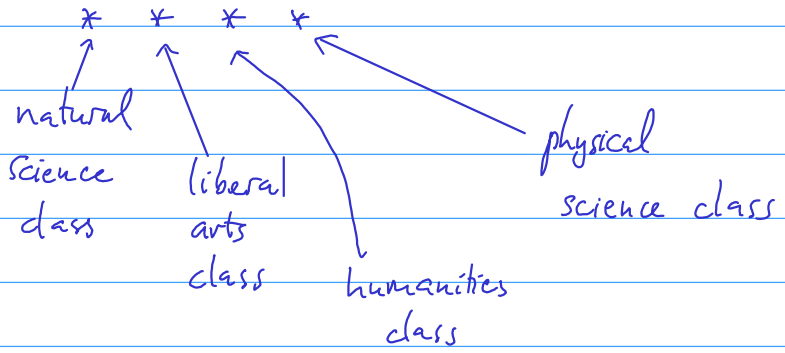


Step 1: choose a jacket → 6 ways
Step 2: choose a pair of pants → 4 ways
Step 3: choose a sweater → 3 ways
Step 4: choose a shirt → 10 ways

The total number of ways to make an outfit is $6 \times 4 \times 3 \times 10 = 720$. That is the number of possible outfits.

Problem 5 of 1.1

- Task: make a schedule of the form

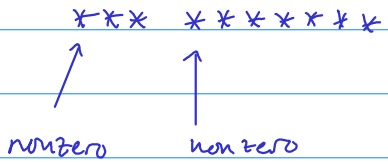


- Step 1: choose natural science class → 3 ways
- Step 2: " liberal arts " → 4 ways
- Step 3: " humanities " → 6 ways
- Step 4: " physical science " → 3 ways

Total number of ways to create a schedule $= 3 \times 4 \times 6 \times 3 = 216$.

Problem 11 of 1.1

- Task: create a telephone number of the form



Step 1: choose a digit to put in the 1st slot → 9 ways
Step 2: " " 2nd " → 10 ways
Step 3: " " 3rd " → 10 ways
Step 4: " " 4th " → 9 ways
Step 5: " " 5th " → 10 ways
Step 6: " " 6th " → 10 ways
⋮ ⋮ ⋮ ⋮
Step 10 " " 11th → 10 ways

The total number of ways to make a telephone is $9 \times 10 \times 10 \times 9 \times 10 \times \cdots \times 10$

Problem 4 of 1.2

(a) $3! - 1! = 6 - 1 = 5$

$(3-1)! = 2! = 2$

(b) $4! + 2! = 24 + 2 = 26$

$(4+2)! = 6! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$

(c) $\frac{4!}{2!} = \frac{24}{2} = 12$

$\left(\frac{4}{2}\right)! = 2! = 2$

(d) $3!2! = 6 \times 2 = 12$

$(3 \times 2)! = 6! = 720$

Problem 9 of 1.2

World population $\approx$ 7 billion

$13! = 6,227,020,800$

$n = 14$ will do it.

Additional problem

• Task: make a passcode of the form

* * *

Step 1: choose an order for the characters:

letter digit symbol
letter symbol digit
digit symbol letter
- - - -

} there are 6 of them

Step 2 choose a letter $\longrightarrow$ 26 ways

Step 3 choose a digit $\longrightarrow$ 10 ways

Step 4 choose a symbol $\longrightarrow$ 9 ways

The total number of passcodes that can be made is $6 \times 26 \times 10 \times 9 = 14,040$.