Homework 2

Do the following problems. Make sure to show your explanation. The odd-numbered problems have the answer key in the back of the book.

Problem 2 of 1.2: Compute the following values.

(a) $P(5,1) = \frac{5!}{4!} = 5$

(b) $P(5,2) = 5 \times 4 = 20$

(c) $P(7,3) = 7 \times 6 \times 5 = 210$

(d) $P(5,5) = 5! = 120$

(e) $P(6,0) = 6! = 720$

(f) $P(100,2) = 100 \times 99 = 9900$

Problem 17 of 1.2: How many ways are there to arrange 10 people in a line? Suppose it took 1 minute to rearrange these 10 people in any order you desired. How many years would it take to try out all of the possibilities. [see below]

Problem 26 of 1.2: How many 5 letter words can be formed if no letter is allowed to be used more than once in any word? (A word is any combination of letters - it does not have to be meaningful.) ? [see below]

Problem 30 of 1.2: How many ordered triples of letters are there, taken from the letters A, T, C and G

(a) if repeated letters are allowed? [see below]

(b) if repeated letters are not allowed?

Problem 32 of 1.2: Suppose you wish to arrange 2 math books, 5 chemistry books, and 4 history books on a single bookshelf.

(a) In how many ways can this be done? [see below]

(b) In how many ways can this be done if the math books must come first, then the chemistry books, and finally the history books?

(c) In how many ways can this be done if all of the books of the same subject must be kept together?

Problem 21 of 1.3: Use formula $C(n,k) = \frac{n!}{(n-k)!k!}$ to compute $C(15,14)$.

Problem 23 of 1.3: Use formula $C(n,k) = \frac{n!}{(n-k)!k!}$ to compute $C(20,7)$. $= \frac{15!}{14!1!} = \frac{15!}{14!} = 15$

$$= \frac{20!}{13!7!} = \frac{1 \times 2 \times 3 \times \cdots \times 20}{1 \times 2 \times 3 \times \cdots \times 13 \times 7!}$$

$$= \frac{14 \times 15 \times 16 \times 17 \times 18 \times 19 \times 20}{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7}$$

$$= 77520$$

* Problem 17 of 1.2:

Each arrangement of 10 people in a line is a permutation of 10 people.
The number of arrangements is $10! = 3628800$.
The amount of time it takes is

$$1 \times 3628800 = 3628800 \text{ minutes}$$
$$= \frac{3628800}{60} \text{ hours}$$
$$= \frac{3628800}{60 \times 24} \text{ days}$$
$$= \frac{3628800}{60 \times 24 \times 365} \text{ years}$$
$$\approx 6.9 \text{ years}$$

* Problem 26 of 1.2

A word is an ordered arrangement of 5 letters picked from a list of 26 letters.
Thus, the number of words is

$$P(26,5) = 7893600$$

* Problem 27 of 1.2 (not part of the Homework)

Task: form a word of 5 letter such that q must be followed by u.

* * * * *

Such a word can belong to one of the two categories:

Category 1: no q

Category 2: has q

- Let's count the number of words in category 1.

A word in this category is a permutation of 5 letters chosen from 25 letters. The number of those permutations is $P(25,5)$.

- Let's count the number of words in category 2.

To form a word in this category, we follow the procedure:

Step 1: pick a spot for q.

There are only 4 possible spots for q. (It can't be at the end of the word because it has to be followed by u.)

Once a spot for q is designated, letter u must be placed immediately after q. There are only 3 spots left. We can't use letters q and u again.

Step 2: put 3 letters chosen from the remaining 24 letters into the 3 remaining spots. There are $P(24,3)$ ways.

Thus, the number of words in Category 2 is $4P(24,3)$.

Therefore, the total number of words is

$$P(25,5) + 4P(24,3) = 6424176$$

Problem 30 of 1.2:

(a) Task: form a word $***$, where each slot is either A, T, C or G.

Step 1: pick a letter to put in the 1st slot. There are 4 ways to do so.

Step 2: pick a letter to put in the 2nd slot. There are 4 ways to do so.

Step 3: pick a letter to put in the 3rd slot. There are 4 ways to do so.

Therefore, there are $4 \times 4 \times 4 = 64$ words

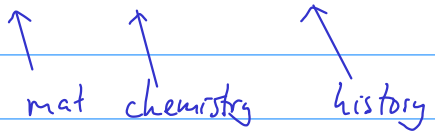
(b) Each triple is an ordered arrangement of A, T, C, G. Therefore, there are $P(4,3) = 24$ words.

Problem 32 of 1.2

(a) Each arrangement on the shelf is an ordered arrangement of 11 books.

Total number of ways is $P(11,11) = 11! = 39916800$

(b) Task: arrange the shelf as MM CCCCC HHHH



Step 1: arrange the math book on the shelf. There are $2! = 2$ ways.

Step 2: arrange the chem book on the shelf. There are $5! = 120$ ways.

Step 3: arrange the hist. book on the shelf. There are $4! = 24$ ways.

The total number of arrangement is $2 \times 120 \times 24 = 5760$ ways

(c) Task: ....

Step 1: choose an order for the book category. There are 6 of them.

MHC CMH HMC

MCH CHM HCM

Step 2: put in the math books. There are $2! = 2$ ways.
Step 3: put in the chem books. There are $5! = 120$ ways
Step 4: put in the hist. books. There are $4! = 24$ ways

Therefore, there are $6 \times 2 \times 120 \times 24 = 34560$ ways