Homework 3 – solution

1) Choose an unordered list of 5 out of 55 : there are $C(55,5)$ ways

$$C(55,5) = \frac{55\times 51\times 50\times 49\times 48}{1\times 2\times 3\times 4\times 5} = \ldots$$

2) (a) Task: make an arrangement $*\,*\,*\,*\,*\,*\,*$ where each $*$ can be either H or T.

Step 1 : choose H or T for the first slot : 2 ways
Step 2 : choose H or T for the second slot : 2 ways
- - - - - - -
Step 7 : choose H or T for the last slot : 2 ways

Thus, there are $2^7 = 128$ ways

(b) To make an arrangement of $*\,*\,*\,*\,*\,*\,*$ in which exactly 3 stars are H, we pick 3 slots out of 7 slots. Then fill in the 3 slots with H, the rest with T. The number of ways to pick 3 slots from 7 slots is $C(7,3)$.

$$C(7,3) = \frac{7\times 6\times 5}{1\times 2\times 3} = 35$$

3) The 4 guesses are either friends of the husband only or friends of the wife only. In the first case, there are $C(8,4)$ ways of choosing 4 friends out of 8 friends to make a party. In the second case, there are $C(9,4)$ ways of choosing 4 friends out of 9 friends of the wife to make a party. Therefore, the total number of ways is

$$C(8,4) + C(9,4) = 196$$

4) (a) When no distinction between the genders is considered, a committee is a group of 3 people chosen from the total of 11 people. There are

$C(11,3) = \frac{11 \times 10 \times 9}{1 \times 2 \times 3} = \ldots$ ways

(b) Task: form a committee of 3 men and 4 women.

Step 1: choose 3 men out of 5 $\longrightarrow$ there are $C(5,3)$ ways.

Step 2: choose 4 women out of 6 $\longrightarrow$ there are $C(6,4)$ ways.

Thus, there are

$$C(5,3) \times C(6,4) = 10 \times 15 = 150 \text{ ways}$$

5) (a) There are $4 \times 3 = 12$ face cards.

Probability is $\frac{12}{52} = \frac{3}{13}$

(b) There are 8 cards that are either king or queen.

Probability is $\frac{8}{52} = \frac{2}{13}$

(c) There are 8 red cards with face value at least 10: 2 tens, 2 jacks, 2 queens, 2 kings.

There are 12 back cards with face value at most 6: 2 aces, 2 twos, 2 threes, 2 fours, 2 fives, 2 sixes.

Probability $= \frac{8+12}{52.} = \frac{20}{52} = \frac{5}{13}$.

6) Sample space $= \{1,2,3,4,5,6\}$.

(a) Prob. of getting 6 is $\frac{1}{6}$.

(b) There are 3 sides with value ≥ 4: 4,5,6

Prob. of getting those is $\frac{3}{6} = \frac{1}{2}$

(c) Event = $\{1, 5, 6\}$

Probability = $\frac{3}{6} = \frac{1}{2}$

7) Sample space = $\{11, 12, 13, 14, 15, 16,$
$21, 22, 23, 24, 25, 26,$
$\ldots\ldots$
$61, 62, 63, 64, 65, 66\}$

(a) Event that sum equals 5 is $\{14, 23, 32, 41\}$.

$P(E) = \frac{4}{36} = \frac{1}{9}$

(b) $E = \{56, 65\}$

$P(E) = \frac{2}{36} = \frac{1}{18}$

(c) $E = \{11, 12, 13, 21, 22\}$

$P(E) = \frac{5}{36}$