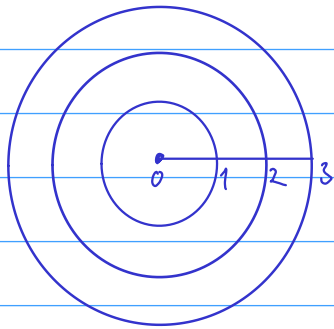


Homework 4 - Solution

1)



Area of the inner-most circle is $\pi 1^2 = \pi$.

Area of the inner ring is $\pi 2^2 - \pi 1^2 = 3\pi$.

Area of the outer ring is $\pi 3^2 - \pi 2^2 = 5\pi$.

Total area = $\pi + 3\pi + 5\pi = 9\pi$.

$$P(\text{dart falling in the inner-most circle}) = \frac{\pi}{9\pi} = \frac{1}{9}$$

$$P(\text{dart falling in the inner ring}) = \frac{3\pi}{9\pi} = \frac{1}{3}$$

$$P(\text{dart falling in the outer ring}) = \frac{5\pi}{9\pi} = \frac{5}{9}$$

2) Sample space = combinations of 5 taken from 52.

Sample size = $C(52, 5)$.

Event = getting 2 Q and 3 K.

To count the number of outcomes constituting an event, we proceed as follows:

Task: create a combination of 2 Q and 3 K

Step 1: pick 2 Q out of 4 Q $\rightsquigarrow C(4, 2)$ ways

Step 2: pick 3 K out of 4 K $\rightsquigarrow C(4, 3)$ ways

The number of ways (= event size) = $C(4, 2)C(4, 3)$.

Probability is $\frac{C(4, 2)C(4, 3)}{C(52, 5)} = \dots$

3) There are 4^{10} possible answer sheets.

E = get at most 9 correct answers.

$E^c =$ getting 10 correct answers (there is only one such an answer sheet).

$$P(E) = 1 - P(E^c) = 1 - \frac{1}{4^{10}} \approx \dots$$

4) Sample set = $\{11, 12, 13, 14, 15, 16,$
 $21, 22, 23, 24, 25, 26,$

 $61, 62, 63, 64, 65, 66\}$

Sample size = 36

$E =$ sum not equal 7

$E^c =$ sum equal 7 = $\{16, 25, 34, 43, 52, 61\}$.

$$P(E) = 1 - P(E^c) = 1 - \frac{6}{36} = \frac{5}{6}$$

5) Sample space = set of 52 cards

Sample size = 52

Event = getting club or getting an even numbered card

$E = A \cup B$

$$P(E) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{13}{52} + \frac{40}{52} - \frac{10}{52} = \frac{43}{52}$$

6) Sample size = 36

$E =$ getting equal side values or getting sum = 6.

$$A = \{11, 22, 33, 44, 55, 66\} \rightarrow P(A) = \frac{1}{6}$$

$$B = \{15, 24, 33, 44, 55, 66\} \rightarrow P(B) = \frac{1}{6}$$

$$A \cap B = \{(3,3)\}$$

$$P(E) = P(A \cap B) = P(A) + P(B) - P(A \cap B) = \frac{1}{6} + \frac{1}{6} - \frac{1}{36} = \frac{11}{36}$$

7) Sample space = {H red, ...
T black

Sample size = 52.

$$E = \text{getting head.} \quad P(E) = \frac{26}{52} = \frac{1}{2}$$

$$F = \text{getting ace.} \quad P(F) = \frac{4}{52} = \frac{1}{13}$$

$E \cap F$ = getting head and ace
= getting head and red ace
= {A heart, A diamond}

$$P(E \cap F) = \frac{2}{52} = \frac{1}{26}$$

Because $P(E \cap F) = \frac{1}{26} = P(E)P(F)$, we conclude that E and F are independent events.