

Homework 6

- 1) Denote by G the event of getting a green ball,
" Y " " yellow " ,
" H " " head,
" T " " tail.

By the law of total probability,

$$\begin{aligned} P(G) &= P(G|H)P(H) + P(G|T)P(T) \\ &= \frac{4}{8} \times \frac{3}{4} + \frac{5}{15} \times \frac{1}{4} = \frac{11}{24} \end{aligned}$$

$$P(Y) = 1 - P(G) = 1 - \frac{11}{24} = \frac{13}{24}$$

- 2) X = amount of money gained from each play.

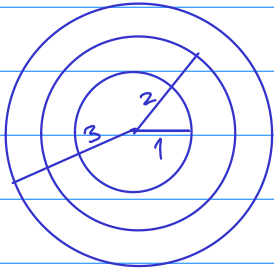
We want to compare EX with 0.

All possible values of X are: $0-1 = -1$ and $2-1 = 1$.

$$\begin{aligned} EX &= (-1)P(X=-1) + (1)P(X=1) \\ &= (-1)P(\text{even face}) + (1)P(\text{odd face}) \\ &= (-1)\frac{3}{6} + (1)\frac{3}{6} \\ &= 0 \end{aligned}$$

This is a fair game. I wouldn't play it because there is, in a long run, no gain for me. I would play if $EX > 0$.

3)



X = amount of money gained from a play.

All possible values of X are:

$$3 - 1.5 = 1.5,$$

$$2 - 1.5 = 0.5,$$

$$1 - 1.5 = -0.5.$$

We want to compare EX with 0.

$$EX = 1.5 P(X=1.5) + 0.5 P(X=0.5) + (-0.5) P(X=-0.5)$$

$$P(X=1.5) = P(\text{hitting center circle}) = \frac{\text{area of inner circle}}{\text{area of outer circle}} = \frac{1^2 \pi}{3^2 \pi} = \frac{1}{9}$$

$$P(X=0.5) = P(\text{hitting middle ring}) = \frac{\text{area of middle ring}}{\text{area of outer circle}} = \frac{2^2 \pi - 1^2 \pi}{3^2 \pi} = \frac{1}{3}$$

$$P(X=-0.5) = P(\text{hitting outer ring}) = \frac{\text{area of outer ring}}{\text{area of outer circle}} = \frac{3^2 \pi - 2^2 \pi}{3^2 \pi} = \frac{5}{9}$$

Therefore,

$$EX = 1.5 \times \frac{1}{9} + 0.5 \times \frac{1}{3} + (-0.5) \times \frac{5}{9} = \frac{1}{18}$$

I see that in a long run, I will win by $\frac{1}{18}$ of a dollar (on average) each time I play. I would play this game.