

Homework 7

$$1) (x^{4/3} y^{8/3})^{3/4} = (x^{4/3})^{3/4} (y^{8/3})^{3/4} = x^{4/3 \times 3/4} y^{8/3 \times 3/4} = x y^2$$

$$2) \frac{\ln 5}{(\ln 2)(\ln 4)} \approx \frac{1.60943}{6.693147 \times 1.38629} \approx 1.674917$$

$$3) \frac{2^k}{12} = 100 \implies 2^k = 1200 \implies k = \log_2 1200 = \frac{\ln 1200}{\ln 2} \\ \approx \frac{7.09007}{6.693147} \approx 10.22881$$

$$4) \text{ Monthly interest rate: } r = \frac{12\%}{12} = 1\% = 0.01$$

$$(a) \text{ After 1 month, we will have } 1000(1+r) = 1000 \times 1.01 = 1010.$$

$$(b) \text{ After 6 months, we will have } 1000(1+r)^6 = 1000 \times 1.01^6 \approx 1061.52$$

$$(c) \text{ After 1 year, we will have } 1000(1+r)^{12} = 1000 \times 1.01^{12} \approx 1126.83$$

$$(d) \text{ After 5 years, we will have } 1000(1+r)^{60} = 1000 \times 1.01^{60} \approx 1816.70$$

$$5) (a) \text{ If compounded annually, then the periodic interest rate is } r = 9.2\% \\ = 0.092.$$

$$\text{\$2000 will grow into } 2000(1+0.092)^5 = 2000 \times 1.092^5 \approx 3105.58$$

$$(b) \text{ If compounded quarterly, then the periodic interest rate is}$$

$$r = \frac{9.2\%}{4} = 2.3\% = 0.023$$

$$\text{\$2000 will grow into } 2000(1+0.023)^{20} \approx 3151.68$$

(c) Monthly: $r = \frac{9.2\%}{12} \approx 0.007666$

\$2000 will grow to $2000(1+r)^{60} \approx 3162.47$

(d) Daily: $r = \frac{9.2\%}{365} \approx 0.00025$

\$2000 will grow to $2000(1+r)^{365 \times 5} \approx 2000 \times 1.00025^{1825} \approx 3156.11$

(e) Continuously: $r = 9.2\% = 0.092$

\$2000 will grow to $2000 e^{r \cdot t} = 2000 e^{0.092 \times 5} = 2000 e^{0.46} \approx 3168.15$

b) $r = \frac{12\%}{4} = 3\% = 0.03$

In t quarters, \$2000 will become $2000(1+0.03)^t = 2000 \times 1.03^t$.

Set

$$2000 \times 1.03^t = 2500$$

$$\Rightarrow 1.03^t = \frac{2500}{2000} = 1.25$$

$$\Rightarrow t = \log_{1.03} 1.25 = \frac{\ln 1.25}{\ln 1.03} \approx 7.55 \text{ quarters} \approx 1.887 \text{ years.}$$