Homework 8 solution

Problem 19 of 2.1: Let r be the interest rate. Then the periodic interest rate is $r/4$. There are 8 quarters in 2 years. Thus, in 2 years, $\$1000$ will grow into

$$1000\left(1+\frac{r}{4}\right)^8 = 1150$$

Divide by 1000 on both sides:

$$\left(1+\frac{r}{4}\right)^8 = 1.15$$

Take the eightth root of both sides:

$$1+\frac{r}{4} = \sqrt[8]{1.15}$$

Subtract 1 from both sides:

$$\frac{r}{4} = \sqrt[8]{1.15} - 1$$

Multiply both sides by 4:

$$r = 4\left(\sqrt[8]{1.15} - 1\right) \approx 0.070495 \approx 7.05\%$$

Problem 22 of 2.1:

Bank 1: 12% interest, compounded quarterly. The periodic interest rate is $r = 12\%/4 = 3\% = 0.03$.

In one year, a loan amount of P will be $P(1+0.03)^4$. The APR is

$$\frac{P(1+0.03)^4 - P}{P} = \frac{P\left((1+0.03)^4 - 1\right)}{P} = (1+0.03)^4 - 1 \approx 0.1255 \approx 12.55\%$$

Bank 2: 11.8% interest, compounded continuously. The periodic interest rate is $r = 0.118$. In one year, a loan amount of P will be $Pe^{r\cdot 1} = Pe^{0.118}$. The APR is

$$\frac{Pe^{0.118} - P}{P} = \frac{P\left(e^{0.118} - 1\right)}{P} = e^{0.118} - 1 \approx 0.125244 \approx 12.52\%$$

Bank 2 is the better choice!

Problem 19 of 2.2:

$P = \$400$ is the monthly payment.

$n = 36$ is the number of periods of payment.

$r = 12\%/12 = 1\% = 0.01$ is the periodic interest rate.

By Theorem 2 on page 29, the present and future values of this annuity are

$$PV = \frac{P}{r}\left(1 - \left(\frac{1}{1+r}\right)^n\right) = \frac{400}{0.01}\left(1 - \left(\frac{1}{1.01}\right)^{36}\right) \approx 12043$$

$$FV = \frac{P}{r}\left((1+r)^n - 1\right) = \frac{400}{0.01}\left(1.01^{36} - 1\right) \approx 17230$$

Problem 37 of 2.2:

First, let's compute the present value of the mortgage (i.e. the principal of the loan).

$P = 1100,\ r = 9\%/12 = 0.0075,\ n = 30 \times 12 = 360$

By Theorem 2 on page 29, the present value of the mortgage is

$$PV = \frac{P}{r}\left(1 - \left(\frac{1}{1+r}\right)^n\right) = \frac{1100}{0.0075}\left(1 - \left(\frac{1}{1.0075}\right)^{360}\right) \approx 136710$$

With $\$25000$ as down payment, the most expensive house you can afford is

$$136710 + 25000 = 161710$$

Problem 39 of 2.2:

You need to find the balance of the loan in 4 years. That is how much you owe at that time. There are two methods to do this.

Method 1: (use Theorem 1 on page 42)

$n = 360,\ k = 48, r = 0.005, A = 100000$. The balance of the loan after 4 years is

$$B(48) = 100000\left[1 - \frac{1.005^{48} - 1}{1.005^{360} - 1}\right] \approx 94614$$

Method 2: (longer method)

The balance of the loan in 4 years (or 48 months) is equal to the future value of the principal in 48 months minus the future value of the payment you will have paid in 48 months.

The present value of the loan is $PV = 100000$.

The monthly interest rate is $r = 6\%/12 = 0.5\% = 0.005$.

The future value of the principal in 48 months is

$$PV(1+r)^{48} = 100000 \times 1.005^{48} \approx 127048.92$$

To compute the future value of the payment you will have paid in 48 months, you use Theorem 2 on page 29. For that, you need to first compute the monthly payment.

The number of payments as originally planned is $n = 360$.

By Theorem 2 on page 29,

$$PV = \frac{P}{r}\left(1 - \left(\frac{1}{1+r}\right)^n\right)$$

Thus,

$$P = \frac{r \cdot PV}{1 - \left(\frac{1}{1+r}\right)^n} = \frac{0.005 \times 100000}{1 - \left(\frac{1}{1.005}\right)^{360}} \approx 599.55$$

Now you can use Theorem 2 on page 29 to find the future value of the payment you will have paid in 48 months:

$$FV = \frac{P}{r}\left((1+r)^{48} - 1\right) = \frac{599.55}{0.005}\left(1.005^{48} - 1\right) \approx 32434.35$$

Therefore, in 4 years you still owe

$$127048.92 - 32434.35 \approx 94614$$

Problem 43 of 2.2: we are comparing two methods of saving.

The first method is to put $\$100$ each month in the bank throughout 30 years (or 360 months) at the interest rate 0.006. The second method is to put a large amount of $\$20000$ in the bank at the interest rate of 0.005 and let it grow on its own for 30 years. The first saving method is annuity. The second is just a one-time payment.

In 360 months, the future value of the stream of payments in the first saving method is (according to Theorem 2 on page 29):

$$\frac{P}{r}\left((1+r)^n - 1\right) = \frac{100}{0.006}\left(1.006^{360} - 1\right) \approx 126923$$

In 5 years, the future value of the one-time payment of $\$20000$ is (according to Theorem 1 on page 25)

$$P(1+r)^n = 20000(1+0.005)^{360} \approx 120452$$

The first saving method is better.

The additional problem:

The principal is $A = 100000$.

The monthly interest rate is $r = 6\% / 12 = 0.005$.

The number of payments is $n = 30 \times 12 = 360$.

The monthly payment is (according to Theorem 1 on page 42):

$$P = \frac{Ar}{1 - \left(\frac{1}{1+r}\right)^n} = \frac{100000 \times 0.005}{1 - \frac{1}{1.005^{360}}} \approx 599.55$$

Using Theorem 1 on page 42, we get the amortization schedule:

Month (k)	Amount going to principal P(k)	Amount going to interest I(k)	Balance of the loan B(k)
1	99.55	500.00	99900.40
2	100.05	499.50	99800.40
3	100.55	499.00	99699.90
4	101.05	498.5	99598.80
5	101.56	497.99	99497.20
6	102.06	497.49	99395.20