Lecture 1

Wednesday, September 14, 2022 9:52 PM

Counting & Math Finance

- counting
- probability

Pick randomly a person in the US

California higher chance to pick someone CA

Pick a random location on earth.

Counting techniques

Pay attention to how a result is created

(2) Ex have 0, 1, 2, ..., 9

For a 3-digits number

* * *

Step1: pick the first slot : 9

Step2: " " : 10

Step3: " " : 10

(3) Pick a number of 3 digits such that * * * the middle digit is largest.

Fund. Count principal

Sequence of steps : # of ways to accomplish one step doesn't depend on how the previous steps were performed.

④ *** different from each other

⑤ Potluck: main – 3
side – 4
dessert – 5

Permutations: $n! = 1 \cdot 2 \cdot 3 \cdots n$

$5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$

Ordered arrangement of object is called permutation

Size n

$$P(n,k) = n(n-1)\cdots(n-k+1) = \frac{n!}{(n-k)!}$$

Counting ⟷ probability

Ex pick randomly a person in the US

most likely from CA

Counting technique

Ex form 2-digit number

How many ways?

procedure — Step 1: pick 1st digit

Step 2: " 2nd "

* *

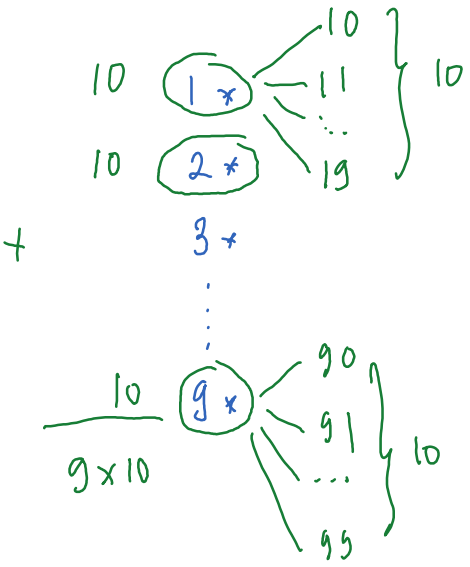
↑ ↑

0, 1, 2, ..., 9

There are 9 ways to do Step 1

There are 10 " " 2

} # of ways to Step1, Step2 is $9 \times 10 = 90$



Ex how many ways to form a 2-digit number such that 1st digit $\leq$ 2nd digit

12 ✓
25 ✓
33 ✓
31 ✗

* *
↓
1, ..., 9

Step 1 : pick 1st digit
Step 2 : " 2nd "

Step 1 : 9 ways
Step 2 : ??

1	1, 2, ..., 9	9
2	2, 3, ..., 9	8

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* Fundamental Counting principle:

Suppose we want to accomplish a task T

Suppose task T can be broken into n steps: $T_1, T_2, T_3, \ldots, T_n$

Suppose there are k_1 ways to do T_1 ←

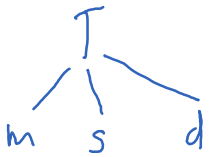
k_2	T_2
⋮	⋮
k_n	T_n

[Suppose the # of ways to do a step doesn't depend on how previous steps are done.]

Then # ways to accomplish T is $k_1 k_2 \ldots k_n$.

Ex Potluck

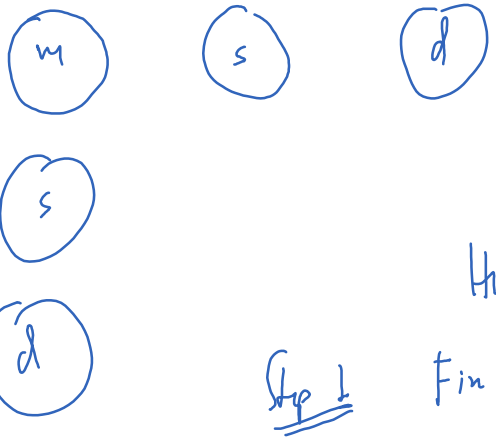
How many ways can we put together a meal?



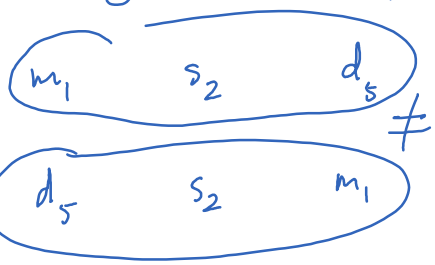
Step 1: pick main dish — 3
Step 2: pick side dish — 4
Step 3: " dessert — 5

} F.C.P $3 \times 4 \times 5 = 60$ ways

Ex



How many meals can be formed?



How to break into steps?

Step 1 Fix an order:

m s d	mds
s m d	sdm
d m s	dsm

6 ways

Step 2 Pick main dish — 3
Step 3: Pick side — 4
Step 4: dessert — 5

Total: $6 \times 3 \times 4 \times 5 = 360$

$n! = 1 \cdot 2 \cdot 3 \cdots n$ factorial

$3! = 1 \cdot 2 \cdot 3 = 6$

$4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$