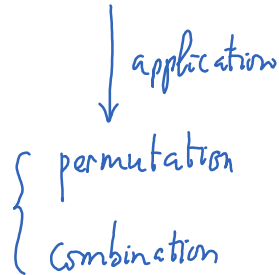


Lecture 2

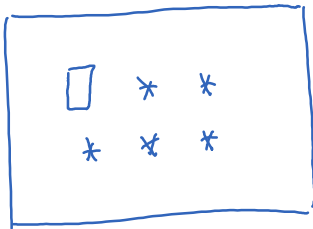
Thursday, October 6, 2022 12:28 AM

Counting techniques

Last time: Fundamental Counting principle



Ex



6 picture frames

1, 2, 3, 4, 5, 6

Task put 6 frames into 6 slots

How many ways?

Step 1: decide what to put on 1st slot ← 6

Step 2: " 2nd slot ← 5

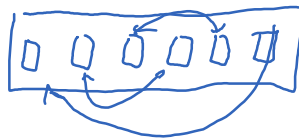
Step 3: " 3rd " ← 4

Step 4: " 4th " ← 3

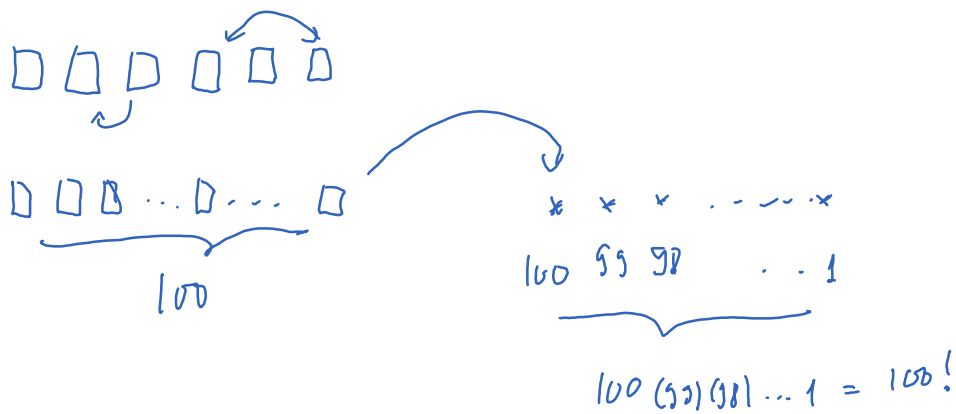
Step 5: " 5th " ← 2

Step 6: " 6th slot ← 1

$$\# \text{ ways} = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = \underline{6!} = 720.$$



A permutation is an ordered arrangement of a collection of objects



of permutations of n objects is $n!$

Ex



10 picture frames

Task: arrange pictures on the wall

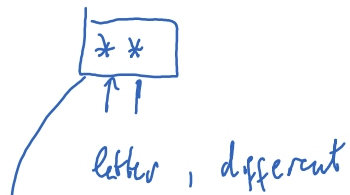
- Step 1: fill in 1st slot $\leftarrow 10$
- Step 2: 2nd $\leftarrow 9$
- Step 3: 3rd $\leftarrow 8$
- Step 4: 4th $\leftarrow 7$
- Step 5: 5th $\leftarrow 6$
- Step 6: 6th $\leftarrow 5$

$$\begin{aligned}
 & 10(9)(8)(7)(6)(5) \\
 &= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1} \\
 &= \frac{10!}{4!}
 \end{aligned}$$

An ordered arrangement of 6 obj. from 10 obj. is called a permutation of size 6 from 10 objects.

Ex

Code



a a a b
b b m z

permutation
of 2 obj taken from 26

26

2

$n=26$
 $k=2$

$$26 \times 25 = \frac{26 \times 25 \times \dots \times 1}{24 \times 23 \times \dots \times 1} = \frac{26!}{24!} = \frac{26!}{(26-2)!}$$

of perm. of k obj chosen from n obj is

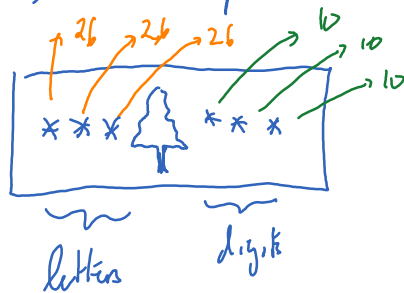
$P(n, k) = \frac{n!}{(n-k)!}$
perm. size of perm.

Ex

$$P(16, 7) = \frac{16!}{(16-7)!} = \frac{16 \times 15 \times \dots \times 7 \times 6 \times \dots \times 1}{1 \times 7 \times 6 \times \dots \times 1} = \underline{\underline{16 \times 15 \times \dots \times 10}}$$

Ex

Oregon's license plate



s1	26	} $26^3 \cdot 10^3$
s2	26	
s3	26	
s4	10	
s5	10	
s6	10	

A A C 123

A B C 779

permutation of 3 dgs chosen from 26

$\boxed{* * *}$

$\boxed{* * *}$

Task: form license plates

Step 1: choose 3 letters from 26 $\rightarrow P(26, 3) = \frac{26!}{(26-3)!} = \frac{26 \times 25 \times 24 \times \dots \times 1}{23 \times 22 \times \dots \times 1}$

Step 2: choose 3 digits from 10 $\rightarrow P(10, 3) = \frac{10!}{(10-3)!} = \frac{10 \times 9 \times 8 \times \dots \times 1}{7 \times 6 \times \dots \times 1} = \boxed{10 \times 9 \times 8} = 720$

Total # license = $\downarrow \times 720 = \dots$