

Lecture 3

Wednesday, October 12, 2022 10:10 PM

Counting techniques

- 1) Fundamental counting principle \rightarrow Task
- 2) Permutation \rightarrow take k objects with order from n obj's $P(n, k)$
- 3) **Combination**

Ex buy 3 chickens among 10 chickens
Task

1, 2, 3, 4, 5, 6, 7, 8, 9, 10

- Step 1: pick 1 chicken among 10 $\rightarrow 10$
- Step 2: pick 1 chicken among 9 $\rightarrow 9$
- Step 3: " " " 8 $\rightarrow 8$

1	5	8
5	1	8
8	1	5
1	8	5
5	8	1
8	5	1

$3! = 1 \times 2 \times 3 = 6$

$10 \times 9 \times 8 = 720$

each set of 3 chickens is repeated 6 times according to the Fund. Count. Principle.

The actual ways of buy 3 chic. from 10 is

$\frac{720}{6} = 120$

A \$ \$A

Combination is an unordered set of objects.

1, A, C, \$, Δ

Permutation

ordered

{A, \$}
 \downarrow
Combination

\$, A

number of perm. of size 2 among 5

$= P(5, 2) = \frac{5!}{(5-2)!}$

$$\{A, \$\} = \{\$, A\}$$

comb.

$$(A, \$) \neq (\$, A)$$

permutation

of combinations of size 2 among 2 obj is $\frac{P(2,2)}{2}$

n objects
pick k of them
don't care about order
how many ways?

= # combinations of size k among n obj

$$= \frac{P(n, k)}{k!}$$

each perm is counted k!

$$= P(n, k) \frac{1}{k!}$$

$$= \frac{n!}{(n-k)!} \frac{1}{k!} = \frac{n!}{(n-k)! k!}$$

$$\frac{n!}{(n-k)! k!}$$

$$C(n, k) = \frac{n!}{(n-k)! k!}$$

Ex

35 questions
At most 7 q wrong

Imagine:

28 q among 35 q.
How many ways?

IIIIIIII CCC . . C
CICICICICICIIICC . . C
1, 2, 3, ..., 35

1, 2, 3, 4, ..., 35

2, 5, 6, 13, 23, 27, 30 = 5, 2, 6, 13, 30, 27

$$\begin{aligned} \# \text{ ways} &= \# \text{ ways to pick 7 out of 35} = C(35, 7) \\ &\quad \uparrow \\ &\quad \text{unordered} = \frac{35!}{(35-7)! 7!} = \frac{35!}{28! 7!} \end{aligned}$$

$$\begin{aligned} \frac{35!}{28! 7!} &= \frac{1 \times 2 \times 3 \times \dots \times 35}{1 \times 2 \times \dots \times 28 \times 1 \times 2 \times \dots \times 7} \\ &= \frac{29 \times 30 \times 31 \times 32 \times 33 \times 34 \times 35}{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7} = \dots \end{aligned}$$

Ex 2 form committee of 7 people

- 3 are faculty among 100 faculty total
- 2 staff among 50 staff total
- 2 academic admin among 20 aca. adm total

How many ways?

Task: form commit of 7 people

FFF SS AA

(James, Mary, Ann)
(Mary, Ann, James)

Step 1 pick 3 fac. among 100 → C(100, 3)

Step 2 pick 2 staff among 50 → C(50, 2)

Step 3 pick 2 adm. among 20 → C(20, 2)

Committees: C(100, 3) × C(50, 2) × C(20, 2)

Probability

{ experiment
 outcome
 sample space: set of all possible outcomes
 event
 probability

pick randomly 1 person on earth

outcome: nationalists

sample space: USA, Russia, Vietnam, Japan, ...

event:

probability of an event \leftrightarrow likelihood

Ex

Experiment: roll 2 dice



outcome: $(1, 1), (1, 2), \dots$

sample space: $(1, 2), (2, 3), \dots$

$(1, 1), (1, 2), (1, 3), \dots, (1, 6)$

$(2, 1), (2, 2), (2, 3), \dots, (2, 6)$

$(3, 1), (3, 2), (3, 3), \dots, (3, 6)$

$(6, 1), (6, 2)$

$(6, 6)$

} 36

$(1, 6), (6, 1)$
 $(2, 5), (5, 2)$
 $(3, 4), (4, 3)$

event: sum of two dice = 1 \rightarrow never happen $\rightarrow P(E) = 0$

sum of two dice = 2 $\rightarrow P(E) = \frac{1}{36}$

sum of two dice = 7 $\rightarrow P(E) = \frac{6}{36} = \frac{1}{6}$

two dice gives the same #

sum < 13 $\rightarrow P(E) = 1$