

Ace

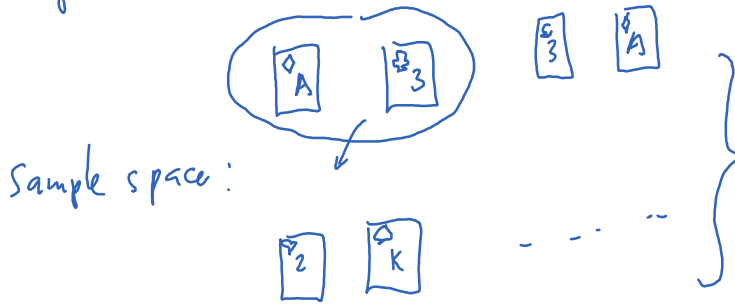
1 2 3

Jack Queen King

11 12 13

E_n

Experiment. draw 2 cards

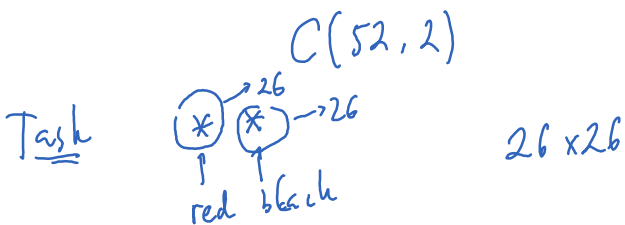


event: 1 red, 1 black

$$\text{probability} = \frac{\text{"size" of event}}{\text{"size" of sample space}} = \frac{26 \times 26}{C(52, 2)}$$

How many poss. outcomes?

* *
choosing 2 cards from 52, no order



Ex Draw 5 cards from deck

Event: there is at least 1 Ace ←

- 1 Ace
- 2 Ace
- 3
- 4
- 5

Outcome: □ □ □ □ □

total # outcome: $C(52, 5)$

Complement event

event E = at least 1 Ace

Comp. event E^c = no Ace

* * * * * 52

with draw 5 cards from 48

$C(48, 5)$

$$P(E) + P(E^c) = 1$$

↓
probability of E

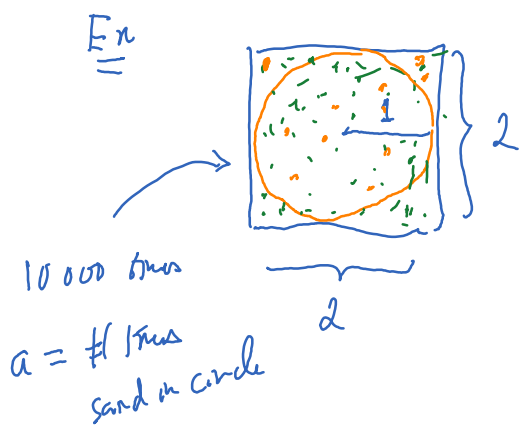
$$P(E^c) = \frac{C(48, 5)}{C(52, 5)} = \frac{\frac{48!}{43! 5!}}{\frac{52!}{47! 5!}} = \frac{48!}{43! 5!} \cdot \frac{47! 5!}{52!}$$

$$= \frac{1 \times 2 \times \dots \times 48}{1 \times 2 \times \dots \times 43} \cdot \frac{1 \times 2 \times \dots \times 47}{1 \times 2 \times \dots \times 52}$$

$$= \frac{44 \times 45 \times 46 \times 47 \times 47}{48 \times 49 \times 50 \times 51 \times 52} \cdot \frac{1}{1}$$

$$\approx 0.66$$

$$P(E) = 1 - P(E^c) \approx 1 - 0.66 = 0.34 = 34\%$$



throw a grain of sand

Event: sand fall inside circle

size of sample space = area of square = $2 \times 2 = 4$

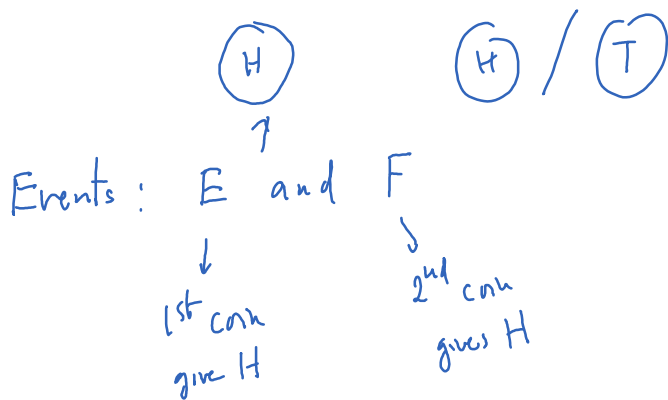
size of event = area of circle = $\pi 1^2 = \pi$

$$\frac{a}{10000} \approx P(E) = \frac{\pi}{4} \quad P(E) = \frac{\pi}{4} \approx \frac{3.14}{4}$$

$$\frac{a}{10000} \times \frac{4}{\pi} \approx \pi$$

* Independence of events:

Toss two fair coins



E_n withdraw 1 card from deck

E = a red card $P(E) = \frac{26}{52} = \frac{1}{2}$

F = a king $P(F) = \frac{4}{52} = \frac{1}{13}$

$$P(E \text{ and } F) = \frac{2}{52} = \frac{1}{26}$$

$$\stackrel{?}{=} P(E)P(F) = \frac{1}{2} \cdot \frac{1}{13}$$

E and F are independent of each other if

$$P(E \text{ and } F) = P(E)P(F)$$

$$E \text{ and } F \iff E \cap F$$

$$E \text{ or } F \iff E \cup F$$

Ex

E = a red card

F = a heart

E & F independent

No!

$$P(E) = \frac{26}{52} = \frac{1}{2}$$

$$P(F) = \frac{13}{52} = \frac{1}{4}$$

$$P(E)P(F) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$$

$$P(E \text{ and } F) = P(\text{red \& heart})$$

$$= P(\text{heart})$$

$$= \frac{13}{52} = \frac{1}{4}$$

