Lecture 5

Wednesday, October 26, 2022 5:57 PM

E, F events : independent if whether E or F happens, that doesn't affect the likelihood of the other.

deck 52 cards.

withdraw 1 card, put back, withdraw 1 card.

E = event that we get Ace in first draw.

F = " Ace " second "

E & F are independent if $P(E \cap F) = P(E)P(F)$

independence $\neq$ disjoint

E & F $\quad P(E \cup F) = P(E) + P(F)$

Ex.

$P(E \cap F) = P$ (getting Ace in both draws)

$= P(E)P(F) = \frac{4}{52} \times \frac{4}{52} = \frac{16}{52 \times 52} = \ldots$

Application of independence

Binomial experiment

"elementary" experiment : 2 outcomes

each outcome has a likelihood (probability)

Ex toss a coin

H / T

↑ ↑

1/2 1/2

Elementary exp. → outcome 1 → success, outcome 2 → unsuccess } Bernoulli experiment / trial

$P(\text{outcome } 1) = p$

$P(\text{out. } 2) = 1-p$

Experiment consisting of many elementary experiments.

Ex

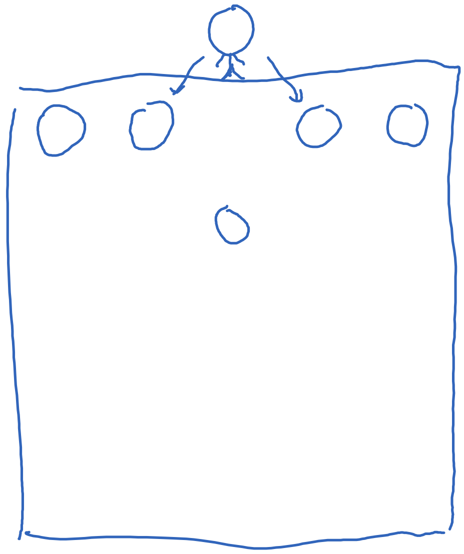
- toss a coin 10 times
- roll a die 5 times, see whether we get face 1

$p = \frac{1}{6}$

$1-p = \frac{5}{6}$

A combination of many Bernoulli exp. that are independent of each other is called a binomial exp.

{roll die once, toss coin twice.}
looking for face 1



pick 1 orange

Do a Bernoulli exp. n times

Count # successes

$\underbrace{0, 1, 2, \ldots, n}_{\text{outcomes}}$

event that we get exactly k successes

probability

$0 \le k \le n$

$$\overset{1-p}{*} \quad \overset{1-p}{*} \quad \overset{p}{*} \quad * \quad \overset{p}{*} \quad \ldots \quad \ldots \quad \overset{p}{*} \quad *$$

$$p(1-p)\cdots(1-p) = p(1-p)^{n-1}$$

n

1 success $= n\,p(1-p)^{n-1}$

k success $=$

$$C(n,k)\, p^k (1-p)^{n-k}$$

$$\boxed{C(n,k)} \quad \underbrace{pp\cdots p}_{k}\ \underbrace{(1-p)(1-p)\cdots(1-p)}_{n-k}$$

In a binomial exp consisting of n Bernoulli exp, the probability of getting k successes is $C(n,k)p^k(1-p)^{n-k}$.

Ex roll a die 10 times

each time, success is when getting 3

probability of getting 5 face 3

$n=10$, $k=5$, $p=\frac{1}{6}$

$C(10,5)\left(\frac{1}{6}\right)^5\left(1-\frac{1}{6}\right)^5$

Conditional prob.

$$\text{prob} = \frac{\text{event size}}{\text{sample size}}$$

Multiple choice: T/F

1) $\leftarrow 2$

2) $\leftarrow 2$

3) $\leftarrow 2$

$2^3 = 8$ possible outcomes.

event of getting 1 T, prob $= \frac{3}{8}$

TTT, TTF, TFT, ...

1) F

2)

3)

TT TF
FT FF

prob $= \frac{2}{4} = \frac{1}{2}$

Def $$\boxed{P(A \mid B) = \frac{P(A \cap B)}{P(B)}}$$

Ex Driver's licence written test: 35 questions A, B, C, D

≥ 28 correct answers to pass.

$$P(\text{passing}) = P(28 \text{ correct}) + P(29 \text{ correct}) + P(30 \text{ correct}) + \dots + P(35 \text{ correct})$$

$$= C(35,28)\, 0.25^{28} (1-0.25)^{7} + C(35,29)\, 0.25^{29} (1-0.25)^{6} + \dots$$

Additional knowledge

Correct Correct Correct Correct $\quad 31$

$$P(\underbrace{\text{passing}}_{A} \mid \underbrace{1^{st}\ 4 \text{ questions correct}}_{B}) = \frac{P(A \cap B)}{P(B)} = \_\_\_\_$$

$\underbrace{*\ *\ *\ *}\quad \underbrace{\qquad 31 \qquad}_{24}$

$$P(B) = \frac{4^{31}}{4^{35}} = \frac{1}{4^4} = \frac{1}{256}$$

$$P(A \cap B) = P(\text{getting} \geq 24 \text{ successes among 31 exp})$$

$$= P(\text{getting 24 succes out of 31}) + P(\text{getting 25 succes out of 31}) + \dots + P(\text{getting 31 succes out of 31})$$

$$= C(31,24)\, 0.25^{24} (1-0.25)^{7} + \dots$$