Lecture 8

Friday, December 2, 2022 11:41 PM

Why interest?

A → B skill make goods

$1000

compensation (interests)

interest → simple interest
interest → compound interest

A = amount of loan

r = interest rate

period	simple interest	compound interest
1	Ar	Ar
2	Ar	$(A+Ar)r = \underline{A(1+r)}r$
3	Ar	$[A+Ar+A(1+r)r]r = Ar(1+r)^2$
4	Ar	$Ar(1+r)^3$

$A+Ar$

interest rate → annual rate / nominal rate / advertised rate
interest rate → periodic interest rate
interest rate → effective interest rate / annual percentage rate (APR)

interest rate
- → annual rate / nominal rate / advertised rate (1) ←
- → periodic interest rate ← (2)
- → effective interest rate / annual percentage rate (APR) (3)

Ex Bank loans money at $\underbrace{\text{12\% interest rate}}_{(1)}$, compound monthly

12% annual rate

$$\underbrace{\text{monthly interest rate}}_{\text{periodic interest rate}} = \frac{12\%}{12} = 1\% = \underline{0.01}$$

loan amount = L

in 12 months : $\text{loan} = L(1+0.01)^{12}$

$$\frac{L(1+0.01)^{12} - L}{L} = \frac{\cancel{L}\left[(1+0.01)^{12} - 1\right]}{\cancel{L}}$$

$$= \boxed{(1+0.01)^{12} - 1}$$

$$= 1.01^{12} - 1 \approx 0.1268$$

$$\approx \underline{12.68\%}$$

effective int rate

annual percentage rate

APR

Ex. Bank A loans money at 6% annual rate, compounded monthly.

Bank B loans money at 5.9% annual rate, compounded quarterly.

Which one to choose?

periodic rate of Bank A $= \dfrac{6\%}{12} = 0.005$

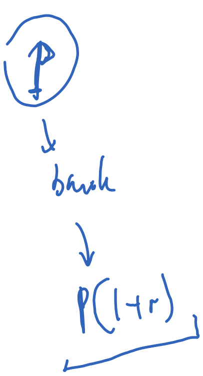
periodic int. rate of Bank B $= \dfrac{5.9\%}{4} = 0.01475$

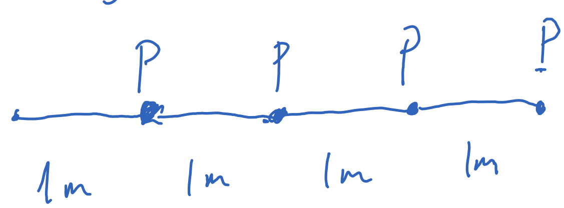
APR for Bank A $= (1+0.005)^{12} - 1 = 6.17\%$

APR for Bank B $= (1+0.01475)^{4} - 1 = 6.03\%$ ✓

---

money ⟷ time





} payment stream
cash flow

Annuity is a payment stream in which each payment period is the same and amount of payment is the same.

value of payment changes over time.

present value   future value

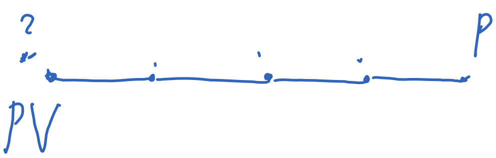
- P, periodic interest rate r

How much P would be in n periods?

$$\boxed{FV = P(1+r)^n} = \text{future value of } P \text{ in } n \text{ periods}$$

- Imagine: after n periods, we have P.

What is the present value



In n period: $PV \to PV(1+r)^n = P$

$$\boxed{PV = \frac{P}{(1+r)^n}}$$

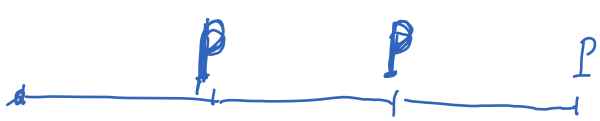
Ex Medical bill = \$300

pay monthly in 3 months

M1 : \$100
M2 : \$100
M3 : \$100

?

period Interest rate = 5% per month



$A = 300$

principal

$P = ?$

$$\frac{P}{1+r} + \frac{P}{(1+r)^2} + \frac{P}{(1+r)^3} = A = 300 \qquad r = 0.05$$

$$P\left[\frac{1}{1+0.05} + \frac{1}{(1+0.05)^2} + \frac{1}{(1+0.05)^3}\right] = 300$$

$$\boxed{P \approx 110.16}$$

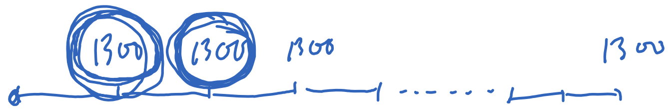
Ex You can afford \$1300/month.

Interest rate $= 6\%$ (annual rate / nominal rate)

loan length $= 30$ years.

How much is the most expensive home you can buy?

A



$\parallel$

principal

$30 \text{ years} = 30 \times 12 = 360$ months

$$r = \frac{6\%}{12} = 0.005$$

$$\frac{1300}{1+r} + \frac{1300}{(1+r)^2} + \frac{1300}{(1+r)^3} + \cdots + \frac{1300}{(1+r)^{360}} = A$$

$P = 1300 =$ monthly payment

$n = 360 =$ # of periods

$$A = \frac{P}{1+r} + \frac{P}{(1+r)^2} + \cdots + \frac{P}{(1+r)^n} \overset{2.3}{=} \frac{P}{r}\left(1 - \frac{1}{(1+r)^n}\right)$$

$$= \frac{1300}{0.005}\left(1 - \frac{1}{(1+0.005)^{360}}\right)$$

$$\approx \boxed{216{,}829}$$

$$\text{loan amount} = 240\,000 - \underbrace{48000}_{} = \underbrace{192\,000}_{\text{loan} = A}$$

$$A = \frac{P}{r}\left(1 - \underbrace{\frac{1}{(1+r)^n}}_{}\right)$$

$$r = \frac{6.386\%}{12}$$

$$P = \frac{Ar}{1 - \frac{1}{(1+r)^n}} \approx \underline{1,199}$$

Amortization schedule

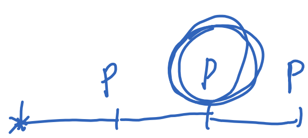
Paying medical bill:

$P = 110.16$

$\boxed{A = 300}$

$\boxed{r = 0.05}$

Ar



k	$P(k)$	$I(k)$	$B(k)$
1			
2			
3			

$P(k)$ = amount of monthly payment that goes into principal at period k

$I(k)$ = ... interest ..

$B(k)$ = balance after k period

$$\begin{cases} P(1) = P - I(1) = \dots \\ I(1) = Ar = \dots \\ \underbrace{B(1)}_{} = A - P(1) = \dots \end{cases}$$

$P(2) = P - I(2) = \dots$

$I(2) = r\,B(1) = \dots$

$B(2) = A - P(1) - P(2) = \dots$