MATH 105, MIDTERM, FALL 2022

INSTRUCTOR: TUAN PHAM

Name

Instructions:

- This is a closed-book exam, 90 minutes long.
- You may bring a 4 x 6 note card (both sides) with any formulas or notes on it. A business, scientific, or graphing calculator is highly recommended. Scratch paper is allowed.
- For Problems 1-11, fill in the bubbles on this front page. To each problem, only one answer is correct.
- For Problems 12, 13, 14, make sure to show all necessary steps. Mysterious answers will receive little or no credit.
- Do not discuss the exam with anyone during Nov 3-8.

1. (A) (B) (C) **(D)**
2. (A) (B) **(C)** (D)
3. (A) **(B)** (C) (D)
4. (A) **(B)** (C) (D)
5. (A) **(B)** (C) (D)
6. **(A)** (B) (C) (D)
7. (A) (B) **(C)** (D)
8. (A) (B) **(C)** (D)
9. (A) **(B)** (C) (D)
10. **(A)** (B) (C) (D)
11. (A) (B) (C) **(D)**

Problem	Possible points	Earned points
1-11	22	
12	5	
13	5	
14	5	
Total	37	

Problem 1. (2 points) Which of the following is equal to $C(6,3)P(7,3)$?

A. 100800
B. 25200
C. 16800
D. 4200

Problem 2. (2 points) Let E and F be two independent events. Which of the following is correct?

A. $P(E \cap F) = 0$

B. $P(E \cup F) = P(E) + P(F)$

C. $P(E \cap F) = P(E)P(F)$

D. $P(E \cup F) = P(E)P(F)$

Problem 3. (2 points) Let E and F be two *disjoint* events such that $P(E) = 1/2$ and $P(F) = 1/6$. Which of the following is the correct value of $P(E \cap F)$ and $P(E \cup F)$?

A. 0 and 1/12

B. 0 and 2/3

C. 1/12 and 2/3

D. 1/12 and 1

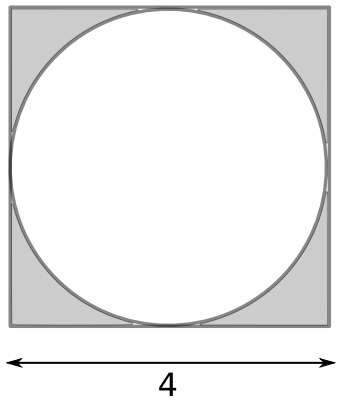
Problem 4. (2 points) Consider a square dart board of side length 4 (shown below). Assume that a dart is equally likely to land on the board at any location. What is the probability that the dart lands outside of the circle?

A. $1 - \pi$

B. $1 - \frac{\pi}{4}$

C. $1 - \frac{\pi}{8}$

D. $\frac{\pi}{16}$



Problem 5. (2 points) To count the number of 2-digit numbers in which the digit on the left is greater than the digit on the right, one solution is as follows:

- Step 1: choose the left digit (9 ways)
- Step 2: choose the right digit (8 ways)
- The total number is $9 \times 8 = 72$.

This solution is

A. Correct
B. Incorrect

Problem 6. (2 points) A student is taking a 5 question true-false test, but does not know any of the answers. How many different possibilities are there for filling out the test?

A. 32
B. 25
C. 16
D. 10

Problem 7. (2 points) A company has 100 employees. How many ways to form a Christmas-party committee consisting of 6 members in which everyone has an equal role ?

A. 100^6

B. $P(100, 6)$

C. $C(100, 6)$

D. 6!

Problem 8. (2 points) How many 3-digit numbers are there whose digits come from 1,2,3,4,5 ?

A. 10
B. 60
C. 125
D. 243

Problem 9. (2 points) A math club has 15 members. How many ways to form a leadership board consisting of 1 president, 1 vice president, 1 secretary, and 1 treasurer (assuming that nobody can play more than one role)?

A. 50625
B. 32760
C. 1365
D. 24

Problem 10. (2 points) Two balls are pick at random from a basket that contains 4 red balls, 3 blue balls, and 2 green balls. What is the probability of getting a red ball and a blue ball?

A. 1/3
B. 1/6
C. 2/3
D. 2/9

Problem 11. (2 points) Two fair dice are rolled. Let E be the event of getting a sum that is at least ten, F be the event of getting a sum that is at most eleven, and H be the event of getting the same faces. Which of the following set of outcomes is equal to the event $E \cup (F \cap H)$?

A. $\{55\}$

B. $\{55, 56, 65\}$

C. $\{11, 22, 33, 44, 55\}$

D. $\{11, 22, 33, 44, 55, 56, 65\}$

Problem 12. (5 points) Two cards are drawn at random from a deck of 52 cards. What is the probability of getting two aces? Explain your answer (for example, your method of counting).

A sample is a combination of size 2 drawn from 52 cards. Thus, the sample size is $C(52,2) = \frac{52 \times 51}{1 \times 2} = 1326$.

Event is a combination of size 2 drawn from 4 cards (the 4 aces). Thus, the event size is $C(4,2) = 6$.

$$\text{probability} = \frac{6}{1326} \approx 0.0045 = 0.45\%$$

Problem 13. (5 points) A card is drawn at random from a deck of 52 cards. Let E be the event of getting a heart, and F be the event of getting a red card. Are E and F independent of each other? Explain using the definition of independence.

$$P(E) = \frac{13}{52} = \frac{1}{4}$$

$$P(F) = \frac{26}{52} = \frac{1}{2}$$

$$P(E \cap F) = P(\text{getting heart and red})$$

$$= P(\text{getting heart}) \qquad (\text{because a heart card is always red})$$

$$= \frac{13}{52} = \frac{1}{4}$$

We see that

$$P(E)\,P(F) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8} \neq P(E \cap F).$$

Therefore, E and F are not independent events.

Problem 14. (5 points) A fair die is rolled 6 times. Find the probability of getting face 4 exactly 2 times. Round your answer up to 4 decimal points. Explain the method you have used.

This is a binomial experiment consisting of 6 Bernoulli experiments.

Probability of success (getting face 4) in each roll is $p = \frac{1}{6}$.

Probability of getting exactly 2 successes in 6 experiments is

$$P(\text{2 successes out of 6 trials}) = C(6,2)\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^4$$

$$= 15 \times \frac{5^4}{6^6} \approx 0.2009 = 20.09\%$$