Homework 1

The following problems are similar to problems in the textbook (pages 14-16), which have solutions (pages 17-19). Feel free to look at those solutions if you need a hint. Also, feel free to make discussion posts on Canvas or look up posts that are already made.

1. To each of the following sets

   $(-1, 5] \cap [0, 4] \cap [1, 6]$,
   $([-2, 4) \cup (5, 7]) \cap [3, 6]$,
   $\{x| -2 < x < 2 \text{ or } x = 3\}$,
   $\{x| -2 < x < 2 \text{ and } x = 3\}$,
   $\{x|x \leq 1 \text{ or } x \neq 3\}$

   do the following:

   (a) Sketch the set on the Real Number line.
   (b) Based on the picture, express the set in interval notation.

2. Consider the points $A(2, 3)$, $B(-3, -2)$, $C(-5, 2)$, $D(4, 0)$, $E(0, 3)$, $F(\sqrt{2}, -\sqrt{3})$.

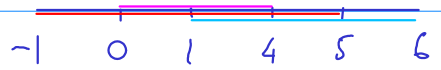
   (a) Plot these points on the Cartesian coordinate plane.
   (b) What quadrants do A, B, C, F lie in?
   (c) Is the triangle ABC an isosceles triangle? If so, what is the apex?
   (d) Find the coordinates of the midpoint of the line segment CD.

3. Find all the points $(1, x)$ which are 5 units from the point $(-2, -3)$.

4. A rational number is any number that can be written as a fraction, where both the numerator and the denominator are integers. How many rational numbers are there in the interval $(0, 1)$? Explain your answer.

Problem 1

$(-1,5] \cap [0,4] \cap [1,6] = [1,4]$



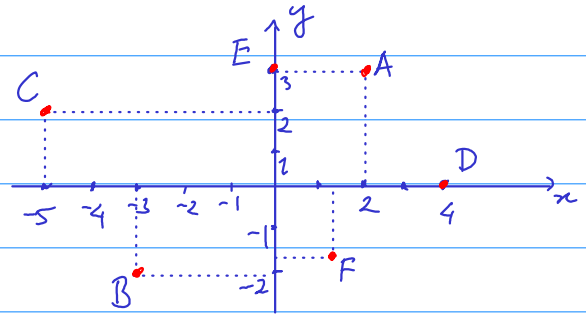
$([2,4) \cup (5,7]) \cap [3,6] = [3,4) \cup (5,6]$

$\{x \mid -2 < x < 2 \text{ or } x = 3\} = (-2,2) \cup [3,3]$

$\{x \mid -2 < x < 2 \text{ and } x = 3\} = \emptyset$ (the empty set)

$\{x \mid x \leq 1 \text{ or } x \neq 3\} = \{x \mid x \neq 3\} = (-\infty, 3) \cup (3, \infty)$

Problem 2



A is in the 1st quadrant,
B in the 3rd quadrant,
C in the 2nd quadrant,
D in the 4th quadrant.

$$AC = \sqrt{(x_C - x_A)^2 + (y_C - y_A)^2} = \sqrt{(-5-2)^2 + (2-3)^2} = \sqrt{50}$$

$$BC = \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2} = \sqrt{(-5-(-3))^2 + (2-(-2))^2} = \sqrt{4+16} = \sqrt{20}$$

$$AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} = \sqrt{(-3-2)^2 + (-2-3)^2} = \sqrt{25+25} = \sqrt{50}$$

We see that $AC = AB$. Thus, ABC is an isosceles triangle with apex at A.

Problem 3

The distance between $(1, x)$ and $(-2, -3)$ is

$$d = \sqrt{(1-(-2))^2 + (x-(-3))^2} = \sqrt{9 + (x+3)^2}$$

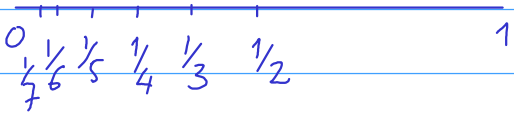
For $d = 5$, we need $9 + (x+3)^2 = 25$.

Thus, $(x+3)^2 = 16 = 4^2$.

This leads to either $x + 3 = 4$ or $x + 3 = -4$.

Therefore, $x = 1$ or $x = -7$.

Problem 4



The rational numbers $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots, \frac{1}{100}, \ldots, \frac{1}{1000}, \ldots$ are between 0 and 1. Therefore, there are infinitely many rational numbers in $(0, 1)$.