Homework 3 - solution

1) (a) Each person has a unique date of birth. Therefore, date of birth is a function of person.

(b) Each phone number corresponds to only one person. A person who has a phone number also has a unique social security number. Thus, social security is a function of phone number. Note: this is not the only explanation. You may argue that a phone number can belong to more than one person (for example, the telephone of a household), each of whom has a different SSN. And therefore, SSN is not a function of phone number.
Both explanations are acceptable.

(c) Each EOU student has a unique student ID, so student ID is a function of the student.

(d) A person may have more than one credit card, so credit card is not a function of the bank customer.

2) (a) Because $2x+3y=4$, we get $y=\frac{1}{3}(4-2x)$. This means one x corresponds to only one y. Thus, y is a function of x.

(b) Because $x^3+y^3=4$, we get $y=\sqrt[3]{4-x^3}$. This means one x corresponds to only one y. Thus, y is a function of x.

(c) Because $y^2=x^3+3x^2$, we get $y=\pm\sqrt{x^3+2x^2}$. A value of x corresponds to two values of y, namely $\sqrt{x^3+2x^2}$ and $-\sqrt{x^3+2x^2}$. Hence, y is not a function of x

3) (a) and (b): y is not a function of x because the graph doesn't pass the vertical line test.

(c) y is a function of x because the graph passes the vertical line test. The domain is $[-1,1]$ and the range is $[0,2]$.

(d) y is a function of x because the graph passes the vertical line test. The domain is $[-2,-1) \cup [0,1]$ and the range is $[-2,0]$.

4)

(a) $x \xrightarrow{\text{square}} x^2 \xrightarrow{\text{add } 5} x^2+5 \xrightarrow{\text{square root}} \sqrt{x^2+5}$

$f(x) = \sqrt{x^2+5}$

Each real value of x gives a unique value of $f(x)$. Thus, the domain is $\mathbb{R}$.

$f(1) = \sqrt{1^2+5} = \sqrt{6}$

$f(2) = \sqrt{2^2+5} = \sqrt{9} = 3$

$f(20) = \sqrt{20^2+5} = \sqrt{405}$

(b) $x \xrightarrow{/3} \frac{x}{3} \xrightarrow{+5} \frac{x}{3}+5 \xrightarrow{\text{square root}} \sqrt{\frac{x}{3}+5}$

$\frac{x}{\sqrt{\frac{x}{3}+5}}$

$f(x) = \frac{x}{\sqrt{\frac{x}{3}+5}}$

* Domain: f is well-defined for any x that makes the denominator not zero.

Domain $= (-15, \infty)$, or $\{x \in \mathbb{R} \mid x > -15\}$

$f(1) = \frac{1}{\sqrt{\frac{1}{3}+5}} = \sqrt{\frac{3}{16}} = \frac{\sqrt{3}}{\sqrt{16}} = \frac{\sqrt{3}}{4}$

$$f(2) = \frac{2}{\sqrt{\frac{2}{3}+5}} = \frac{2}{\sqrt{\frac{17}{3}}} = \frac{2\sqrt{3}}{\sqrt{17}},$$

$$f(20) = \frac{20}{\sqrt{\frac{20}{3}+5}} = \frac{20\sqrt{3}}{\sqrt{35}}.$$