Homework 4 - Solution

1) $f(x) = x^2 + x$

$f(x+h) = (x+h)^2 + (x+h) = (x+h)(x+h) + x + h$

$= x^2 + 2xh + h^2 + x + h$

$f(x+h) - f(x) = 2xh + h^2 + h$

$$\frac{f(x+h) - f(x)}{h} = \frac{2xh + h^2 + h}{h} = 2x + h + 1$$

2) $f(x) = x + 1$, $g(x) = \dfrac{1}{\sqrt{x+2}}$.

(a) $(f+g)(1) = f(1) + g(1) = (1+1) + \dfrac{1}{\sqrt{1+2}} = 2 + \dfrac{1}{\sqrt{3}}$

(b) $\left(\dfrac{f}{g}\right)(-3) = \dfrac{f(-3)}{g(-3)} = \dfrac{-3+1}{\dfrac{1}{\sqrt{-3+2}}}$

We see that $\sqrt{-3+2}$ is undefined. Therefore, $\left(\dfrac{f}{g}\right)(-3)$ is undefined.

(c) $\left(\dfrac{g}{f}\right)(-1) = \dfrac{g(-1)}{f(-1)} = \dfrac{\frac{1}{\sqrt{-1+2}}}{-1+1}$

We see that the denominator is $-1+1 = 0$. Therefore, $\left(\dfrac{g}{f}\right)(-1)$ is undefined.

(d) $(fg)(2) = f(2)g(2) = (2+1)\dfrac{1}{\sqrt{2+2}} = 3\dfrac{1}{\sqrt{4}} = \dfrac{3}{2}$

3)

a) $f(x) = x - 1$. Domain is $\mathbb{R}$.

To find y-intercept, we let $x = 0$. Then $f(0) = 0 - 1 = -1$.

The y-intercept is $(0, -1)$.

To find x-intercept, we let $y = 0$. Then $f(x) = x - 1 = 0$. Then $x = 1$.

The x-intercept is $(1, 0)$.

The function f is neither even or odd because $f(1) = 0$ and $f(-1) = -2$ (and so $f(1) \neq f(-1)$, $f(1) \neq -f(-1)$).

(b) $f(x) = \dfrac{1}{x^2-1}$

To find y-intercept, we let $x = 0$: $f(0) = \dfrac{1}{0^2-1} = -1$.

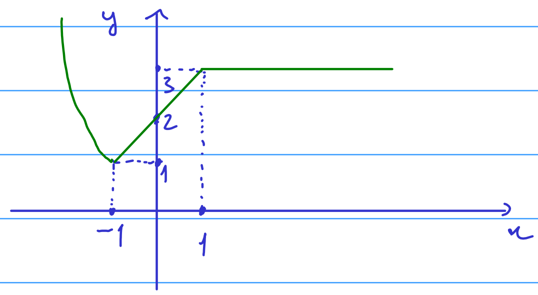
Thus, the y-intercept is $(0,-1)$.
f doesn't have any x-intercept because $f(x)$ is never equal to zero.

$$f(-x) = \frac{1}{(-x)^2-1} = \frac{1}{x^2-1} = f(x)$$

Hence, f is an even function.

4)
$$f(x) = \begin{cases} x^2, & x \leq -1 \\ x+2, & -1 < x \leq 1 \\ 3, & x > 1 \end{cases}$$

(a)



(b)

$f(-1) = 1$, $f(0) = 2$, $f(1) = 3$, $f(2) = 3$.

(c) There is no x such that $f(x) = 0$ because the graph of f doesn't intersect the x-axis.

5) Domain $= \mathbb{R}$
Range $= \mathbb{R}$

$f(x) \geq 0$ for any $x \in \mathbb{R} = (-\infty, \infty)$.

f is decreasing on $(-\infty, -1] \cup [0, 2]$ and increasing on $[-2, 0] \cup (2, \infty)$.

Local minima are $(-2, 0)$ and $(2, 0)$.

Local maximum is $(0, 4)$.