Homework 5

Problem 1 is similar to the example on page 95. Problem 2 is similar to the example on page 127. Problem 3 is similar to the examples on page 152 and 157. Problem 4 is similar to Problem 32 on page 164.

1. Determine analytically if the following functions are even, odd or neither. That is to find $f(-x)$ and then compare it to $f(x)$ or $-f(x)$.

   (a) $f(x) = x^3 + x^2 + 1$

   (b) $f(x) = 1 - x^2$

   (c) $f(x) = x - x^3$

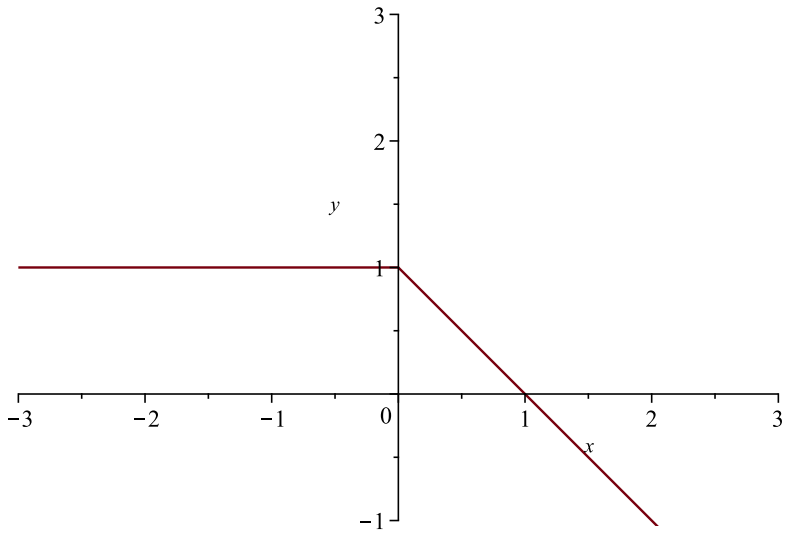
2. The graph of $y = f(x)$ is given below. Graph the following transformed functions.

   (a) $f(x) + 1$

   (b) $f(x + 1)$

   (c) $f(-x + 1)$

   (d) $-2f(x)$



3. Find the point-slope form, the slope-intercept form, the x-intercept, and the y-intercept of the line that

   (a) passes through $P(-1, 1)$ and $Q(1, 2)$;

   (b) passes through $P(-1, 1)$ with slope equal to 2.

4. A fitness trainer is paid \$1,800 a month plus 15% commission on his monthly sales (of personal training contracts) of x dollars. Find a linear function that represents his total monthly pay, called W, in terms of x. What must his monthly sales be in order for him to earn \$3,000 for the month?

1) (a) $f(-x) = (-x)^3 + (-x)^2 + 1 = -x^3 + x^2 + 1$

$f(x) = x^3 + x^2 + 1$

$-f(x) = -x^3 - x^2 - 1$

We see that $f(-x)$ is not equal $f(x)$ or $-f(x)$. Thus, f is not an even or odd function.

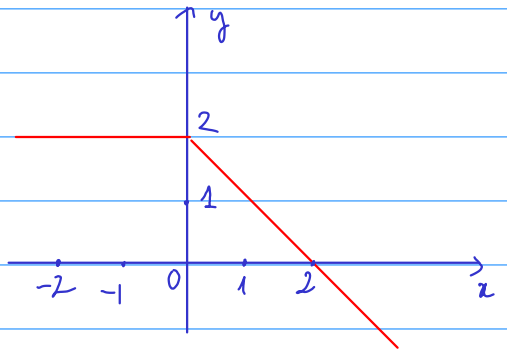
(b) $f(-x) = 1 - (-x)^2 = 1 - x^2 = f(x)$

f is an even function.

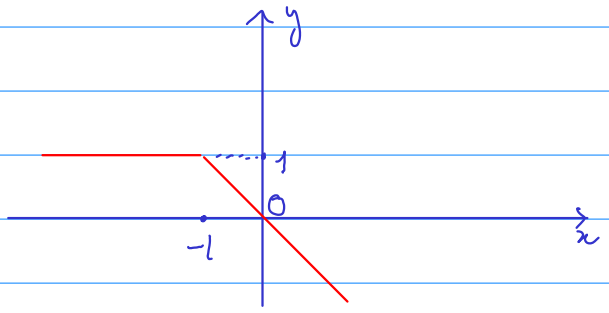
(c) $f(-x) = (-x) - (-x)^3 = -x + x^3 = -(x^3 - x) = -f(x)$

f is an odd function.

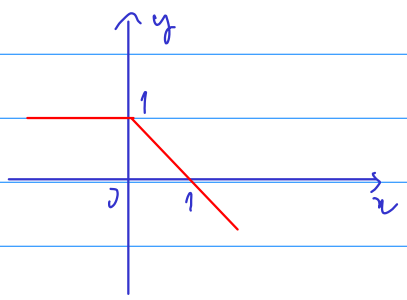
2) (a) Shift the graph up by 1 unit:



(b) Shift 1 unit to the left.

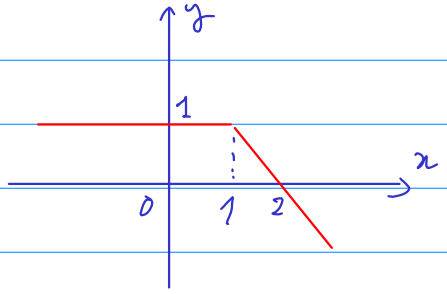


(c) $f(-x+1) = f(-(x-1))$, the graph of this function is the y-axis reflection of the graph of the function $f(x-1)$, which is obtained by shifting the graph of $f(x)$ to the right by 1 unit.



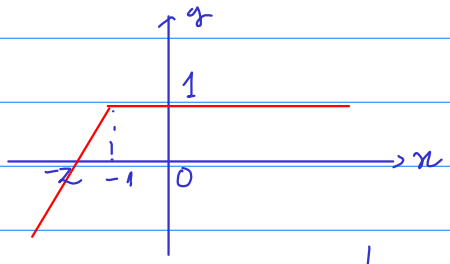
graph of $f(x)$

shift right



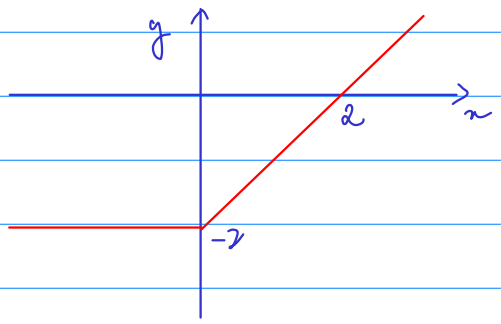
graph of $f(x-1)$

reflection wrt y-axis



graph of $f(-x+1)$

(d)



scale the graph of $f(x)$ by factor 2 and then take the reflection of the graph with respect to the x-axis.

3) (a) line passing through $P(-1,1)$ and $Q(1,2)$ has slope

$$m = \frac{2-1}{1-(-1)} = \frac{1}{2}$$

Equation: $y - 1 = \frac{1}{2}(x-(-1))$ This is the slope form.

Equivalently: $y = \frac{1}{2}x + \frac{3}{2}$ This is the slope-intercept form

y-intercept is $(0, \frac{3}{2})$

x-intercept: solve $\frac{1}{2}x + \frac{3}{2} = 0$ $\rightsquigarrow$ get $x = -3$ $\rightsquigarrow$ x-intercept is $(-3, 0)$.

(b) line passing through $P(-1, 1)$ with slope 2:

$$y - 1 = 2(x - (-1)) \quad \leftarrow \text{point-slope form}$$
$$y = 2x + 3 \quad \leftarrow \text{slope-intercept form}$$

y-intercept: $(0, 3)$

x-intercept: $(-\frac{3}{2}, 0)$

4) $W = 1800 + 0.15x$

For $W = 3000$, we need $0.15x = 3000 - 1800 = 1200$.

Then $x = \frac{1200}{0.15} = 8000$