

Homework 6

1) (a) $|2x+3|=7$

Because $|2x+3|$ is either $2x+3$ or $-(2x+3)$, we have to solve two equations:

$$2x+3=7 \quad (1)$$

$$\text{and } -(2x+3)=7 \quad (2)$$

From equation (1), we get $x=(7-3)/2=2$.

From equation (2), we get $2x+3=-7$, so $x=(-7-3)/2=-5$.

Conclusion: there are two solutions, namely $x=2$ and $x=-5$.

2) $f(x) = |x+2| - |2x-1|$

To remove the absolute value signs, we need to know the signs of $x+2$ and $2x-1$. To do this, we need to compare x with -2 and $\frac{1}{2}$.

x	-2	$\frac{1}{2}$
$ x+2 $	$-x-2$	$x+2$
$ 2x-1 $	$1-2x$	$2x-1$
$f(x)$	$x-3$	$-x+3$

$$f(x) = \begin{cases} x-3 & \text{if } x \leq -2 \\ 3x+1 & \text{if } -2 \leq x \leq \frac{1}{2} \\ -x+3 & \text{if } x \geq \frac{1}{2} \end{cases}$$

To find y -intercept, plug $x=0$ (using the middle equation):

$$f(0) = 3(0)+1 = 1$$

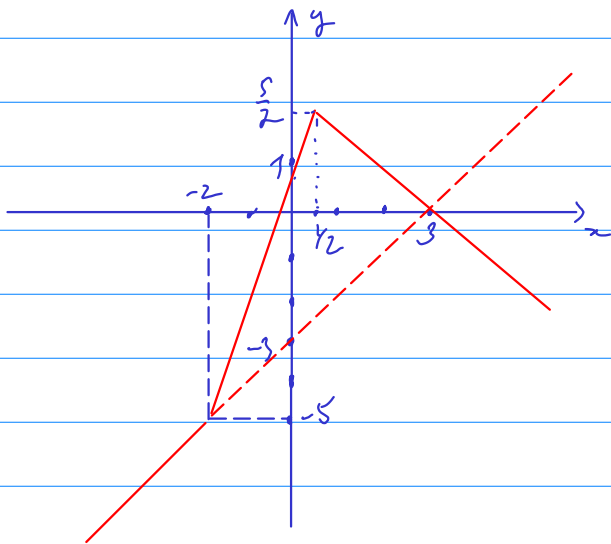
y -intercept is $(0, 1)$.

For x -intercept, we can try different formulae of $f(x)$.

- $x-3=0 \rightarrow x=3$, not acceptable because $3 > -2$
- $3x+1=0 \rightarrow x=-\frac{1}{3}$, acceptable because $-2 \leq -\frac{1}{3} \leq \frac{1}{2}$
- $-x+3=0 \rightarrow x=3$, acceptable because $3 \geq \frac{1}{2}$.

In conclusion, there are two x -intercepts: $(-\frac{1}{3}, 0)$ and $(3, 0)$.

(b)



(c) Domain $= \mathbb{R}$

Range $= (-\infty, \frac{5}{2}]$

(d) Increasing on $(-\infty, \frac{1}{2}]$. Decreasing on $[\frac{1}{2}, \infty)$.

(e) No local minimum.

Local maximum of $\frac{5}{2}$, attained at $x = \frac{1}{2}$.