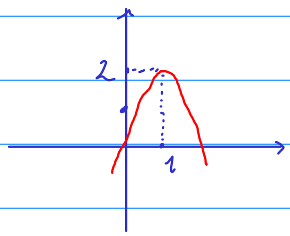
Homework 7

1) (a) $f(x) = -(x-1)^2+2 \rightsquigarrow$ this is already in standard form

Vertex has coordinates $(1,2)$

Axis of symmetry is $x=1$

$$-(x-1)^2+2 = -(x^2-2x+1)+2 = -x^2+2x-1+2 = \underbrace{-x^2+2x+1}_{\text{general form}}$$



Vertex is where f attains maximum.

(b) $f(x) = 2x^2-4x-6 \rightsquigarrow$ this is the general form.

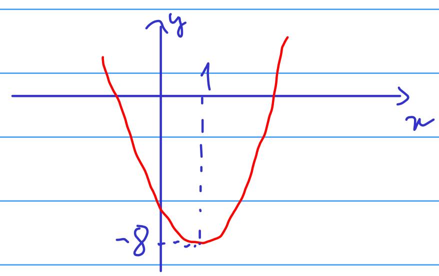
To get the standard form, we complete the square!

$$\begin{aligned} 2x^2-4x-6 &= 2(x^2-2x-3) \\ &= 2(x^2-2x+1-4) \\ &= 2((x-1)^2-4) \\ &= 2(x-1)^2-8 \rightsquigarrow \text{standard form.} \end{aligned}$$

Coordinates of apex is $(1,-8)$.

Axis of symmetry: $x=1$.

Vertex is where f attains minimum.



2) $$T(t) = -\frac{1}{2}t^2 + 8t + 32, \qquad 0 \le t \le 12$$

Let us try to graph the parabola:

$$T(t) = -\frac{1}{2}(t^2 - 16t - 64) = -\frac{1}{2}(t^2 - 16t + 64 - 128)$$

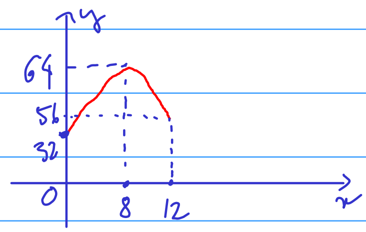
$$= -\frac{1}{2}\left((t-8)^2 - 128\right)$$

$$= -\frac{1}{2}(t-8)^2 + 64 \leftarrow \text{standard form}$$

Vertex is $(8, 64)$.

Axis of symmetry is $t = 8$.

The parabola is facing downward because $-\frac{1}{2} < 0$.

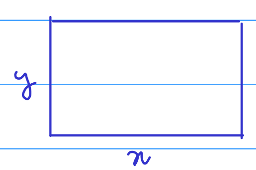


$f(0) = 32$, $f(12) = 56$, $f(8) = 64$

The maximum temperature is $64°F$, attained at $t = 8$.

The minimum value is $32°F$, attained at $t = 0$.

3)



$x + y = 2$

$xy \to \max$?

Area is $xy = x(2-x) = -x^2 + 2x = f(x)$

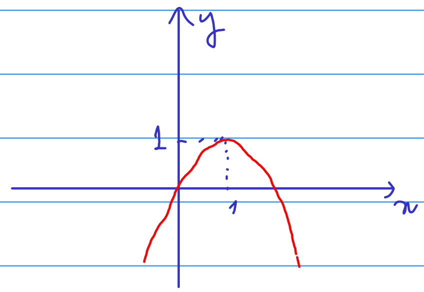
We want to find the value of x such that $f(x)$ is maximum.

Let's convert $f(x)$ into standard form:

$$-x^2 + 2x = -(x^2 - 2x) = -(x^2 - 2x + 1 - 1) = -\left((x-1)^2 - 1\right)$$

$$= -(x-1)^2 + 1$$

Vertex is $(1, 1)$.



Maximum value of $f(x)$ is 1, attained at $x=1$.

Thus, the maximum area of the rectangle is 1, which happens when the sides are 1 (rectangle becomes a square).