Lecture 4

Thursday, October 20, 2022 12:31 AM

Quantity y Quantity x

y is a function of x

$y = y(x)$

$\underline{y = f(x)}$

$x \longrightarrow \boxed{f} \longrightarrow y$

input machine output

function is a relation $\longrightarrow \boxed{\text{graph}}$

1. $f(x) = x^2 + x \longrightarrow$ how to graph?

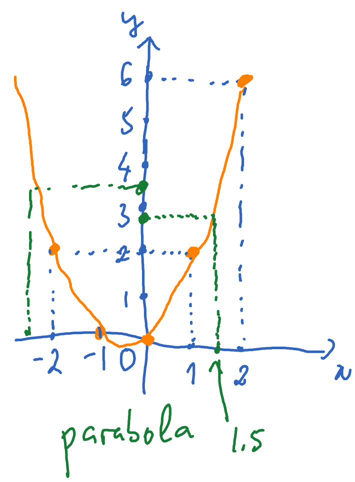
$x \longrightarrow f \longrightarrow x^2 + x$

$0 \longrightarrow f \longrightarrow 0^2 + 0 = 0$

$1 \longrightarrow f \longrightarrow 1^2 + 1 = 1 + 1 = 2$

$f(-1) = (-1)^2 + (-1) = 1 + (-1) = 0$

x	y
0	0
1	2
-1	0
-2	2
2	6



Properties of a function

f is called an <u>even</u> function if $f(-x) = f(x)$ for any x.

$-x \searrow$, $x \nearrow$ $f \longrightarrow y$

f is an odd function if $f(-x) = -f(x)$ for any x.

$$\begin{tikzcd} x \arrow[r] & f \arrow[r] & y \arrow[d, leftrightarrow] \\ -x \arrow[r] & f \arrow[r] & -y \end{tikzcd}$$

Ex

$$f(x) = x^2$$

$$f(-x) = (-x)^2 = (-x)(-x) = x^2 = f(x)$$

$$f(-x) = f(x) \rightarrow f \text{ is an even function}$$

Ex

$$f(x) = \frac{1}{x}$$

$$f(-x) = \frac{1}{-x} = -\frac{1}{x} = -f(x)$$

f is an odd function.

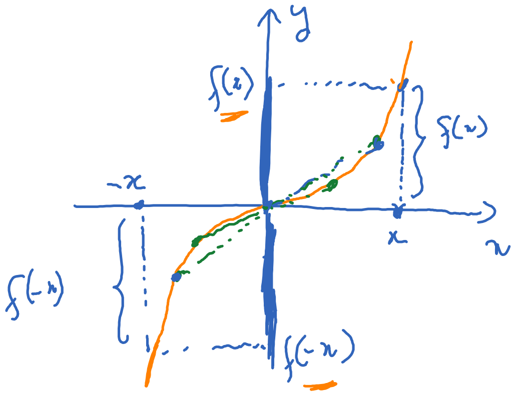
Ex

$$f(x) = x^2 + x$$

$$f(-x) = (-x)^2 + (-x) = x^2 - x$$

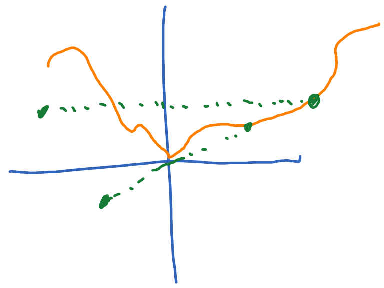
$$-f(x) = -(x^2+x) = -x^2 - x$$

f is neither even or odd.



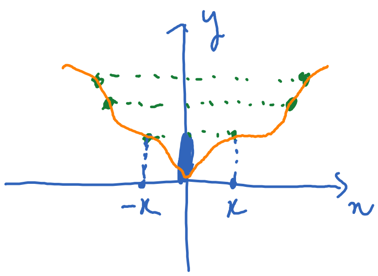
f is odd

the graph of f is symmetric with respect to the origin



not an odd function

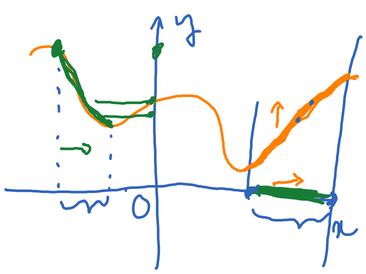
not an even function



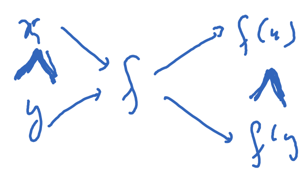
f is even

graph of f is symmetric wrt the y-axis

symmetry properties — even / odd

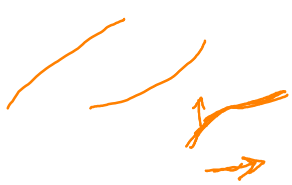


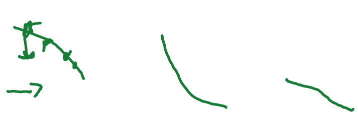
f is increasing on an interval I if $f(x) < f(y)$ for $x < y$.

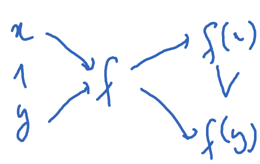


f is decreasing on interval I

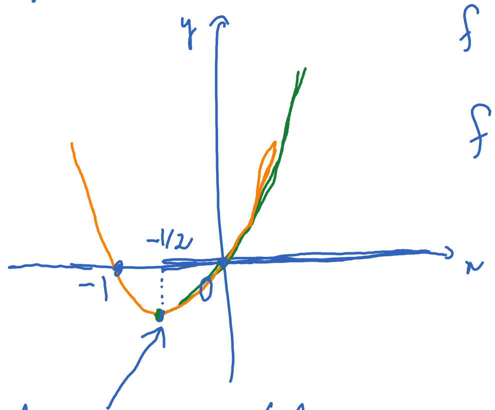
if $f(x) < f(y)$ for $x > y$.







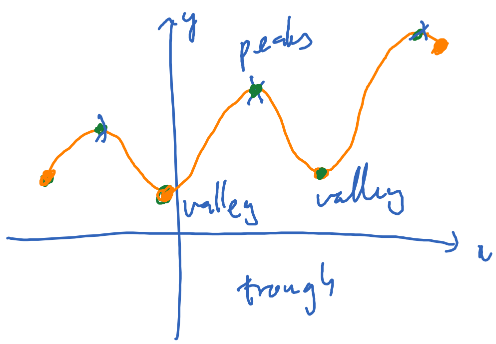
Ex $f(x) = x^2 + x$

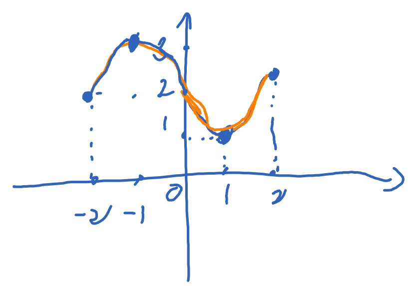


f is increasing on interval $\left(-\frac{1}{2}, \infty\right)$.

f is decreasing on interval $\left(-\infty, -\frac{1}{2}\right)$

local minimum / local maximum.





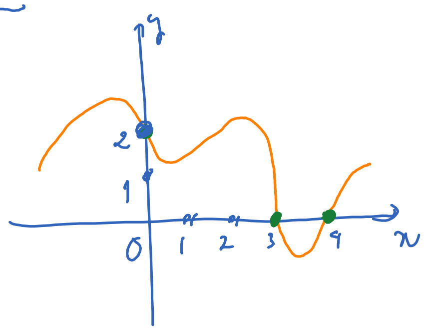
f attains a local minimum at $x = 1$

f " " " at $x = -2$

f " " maximum at $x = -1$

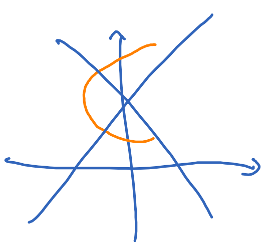
$x = 2$

Intercepts



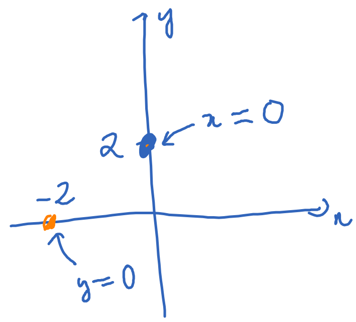
y-intercept is $(0, 2)$

x-intercepts are $(3,0)$, $(4,0)$



Ex $\quad f(x) = \dfrac{x+2}{x+1}$

Find the x-intercepts and y-intercepts.



x-intercept : $\underbrace{f(x)}_{y} = 0$ solve for x

$\underbrace{x}_{\text{a zero of } f} \longrightarrow f \longrightarrow 0$

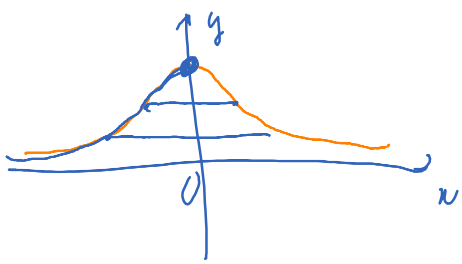
$\dfrac{x+2}{x+1} = 0$, we get $x+2=0$, $\underline{x=-2}$

x-intercept is $(-2,0)$

y-intercept : $x = 0$

$y = f(0) = \dfrac{0+2}{0+1} = 2$

is the point $(0,2)$.



$f(x) = \dfrac{1}{x^2+1}$

even

increasing on $(-\infty, 0]$

decreasing on $[0, \infty)$

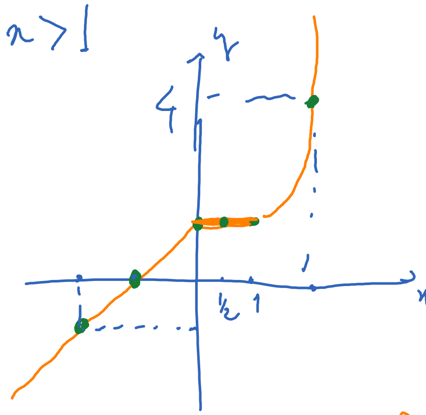
f attains a local max at $x = 0$.

Ex

$$f(x) = \begin{cases} x+1 & \text{if } x < 0 \\ 1 & \text{if } 0 \le x \le 1 \\ x^2 & \text{if } x > 1 \end{cases}$$

$f(-2) = (-2)+1 = -1$

$f\left(\frac{1}{2}\right) = 1$



$f(-1) = (-1)+1 = 0$

$f(0) = 1$

$f(1) = 1$

$f(2) = 2^2 = 4$

f is constant on [0,1]

Algebra on functions

$f(x) = x^2+1$

$g(x) = \frac{1}{x}$

$(fg)(x) = f(x)\,g(x) = (x^2+1)\,\frac{1}{x} = \frac{x^2+1}{x}.$

new function

$x \longrightarrow fg \longrightarrow \frac{x^2+1}{x}$

$(f+g)(1) \overset{?}{=} f(1)+g(1) = (1^2+1) + \frac{1}{1} = 3.$

$(f+g)(x) = f(x)+g(x)$