Lecture 5

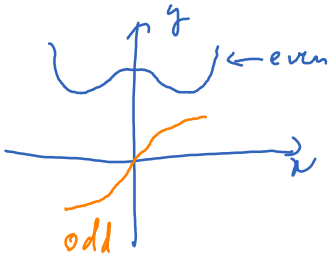
Thursday, October 27, 2022 12:46 AM

Graph of a function $y = f(x)$

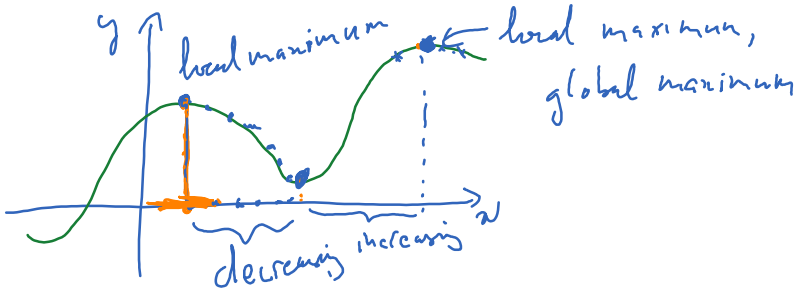
* symmetry: odd / even / neither

$f(-x) = -f(x)$

$f(-x) = f(x)$

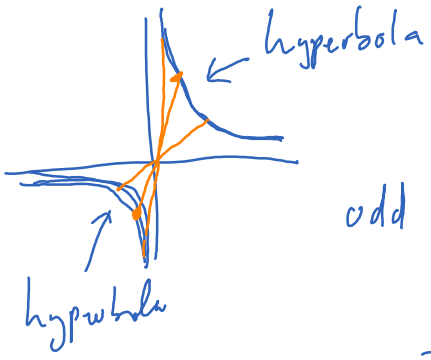


* monotonicity: increasing / decreasing



* extrema { minima, maxima

Ex $f(x) = \frac{1}{x}$



odd

Analytical method

$f(-x) = \frac{1}{-x} = -\left(\frac{1}{x}\right) = -f(x)$

odd

$g(x) = \dfrac{1}{x+1}$

$g(-x) = \dfrac{1}{-x+1}$ vs $-g(x) = -\dfrac{1}{x+1}$

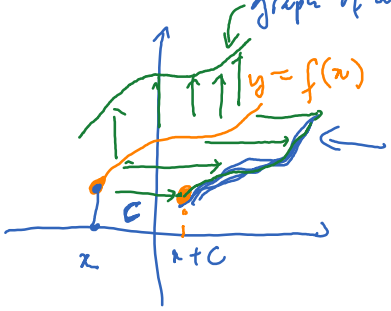
$x=2$: $g(2) = \dfrac{1}{2+1} = \dfrac{1}{3}$ ← not the same

$g(-2) = \dfrac{1}{-2+1} = \dfrac{1}{-1} = -1$

g is not even.

left $\overset{x=2}{=} \dfrac{1}{-2+1} = -1 \neq$ right $\overset{x=2}{=} -\dfrac{1}{2+1} = -\dfrac{1}{3}$

g is not odd.



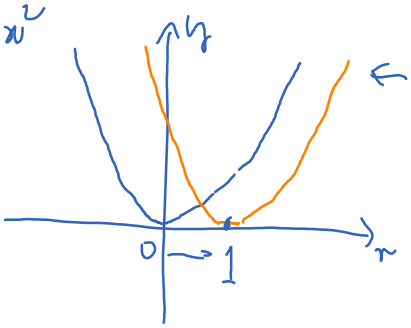
what function? $g(x)$

$x + c = t \rightarrow x = t - c$

$g(\underbrace{x+c}_{t}) = f(x)$

$\boxed{g(t) = f(t-c)}$

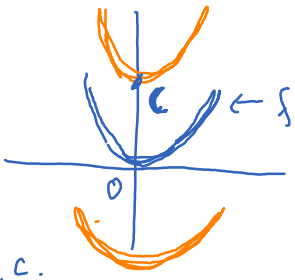
Ex $f(x) = x^2$



← $g(t) = f(t-1) = (t-1)^2$

graph of $f(x)$

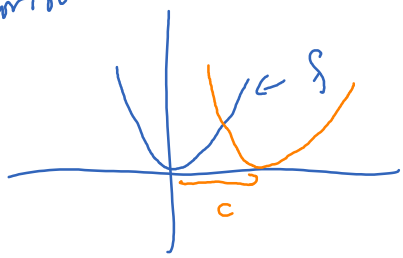
$f(x)+c$: vertical shift by c unit



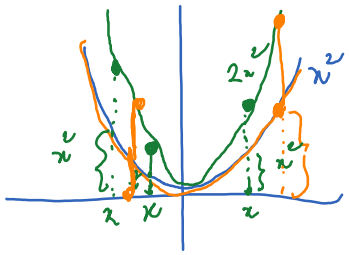
$f(x-c)$: horizontal shift by c unit

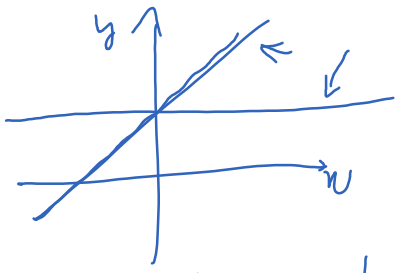
$cf(x)$: scale vertically by factor c

$f(cx)$: scale horizontally by factor of c.

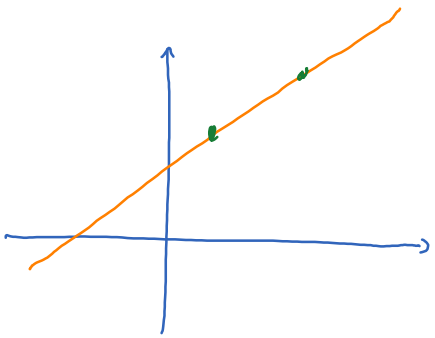


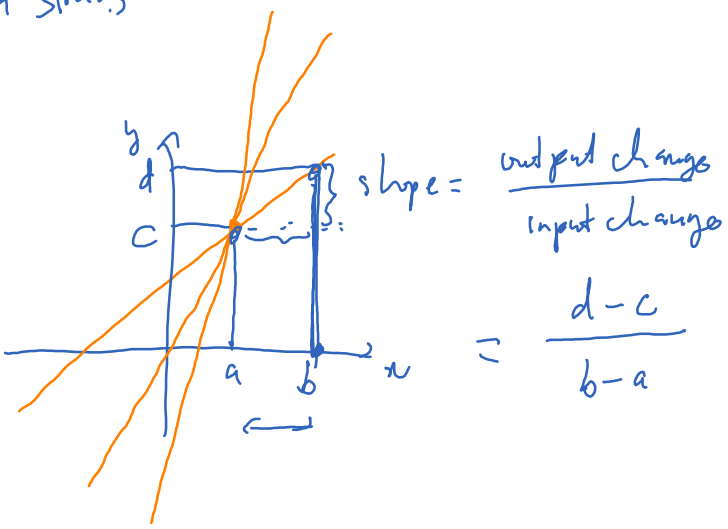
$2x^2$





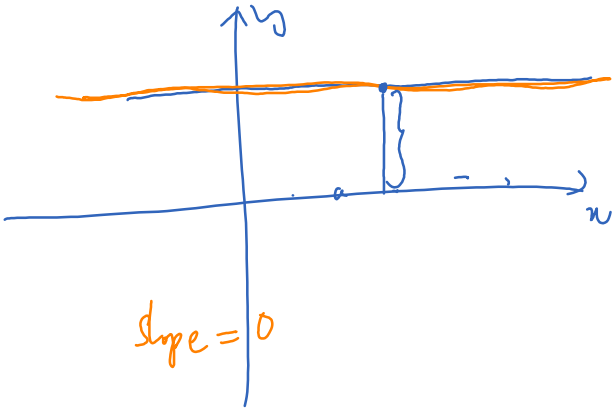
linear function : graph is a straight line





$$\text{slope} = \frac{\text{output change}}{\text{input change}} = \frac{d-c}{b-a}$$

slope $= 0 = \frac{\text{output change}}{\text{input change}}$ output change $= 0$.

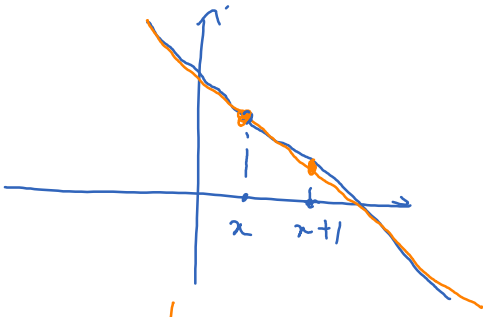


slope $= 0$

slope $= -1 = \frac{\text{out. ch.}}{\text{in. ch.}}$

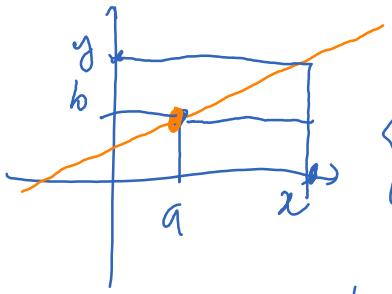
(-1) in. ch. $=$ out ch.

$-\text{input change} = \text{output change}$



slope < 0

What is the formula of a linear function?



slope $= m$ given

$f(a) = b$ given

input change $= x - a$

output " $= y - b$

$\frac{y-b}{x-a} = m$ (given)

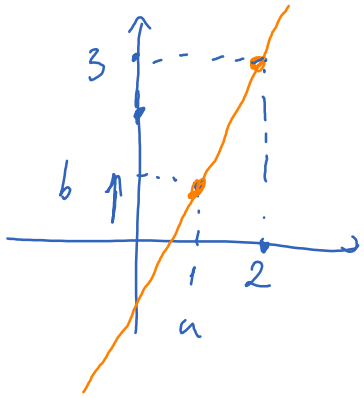
Mult. by $x-a$:

$$y - b = m(x-a)$$

$y = f(x)$

slope-point formula of a linear func.

Ex



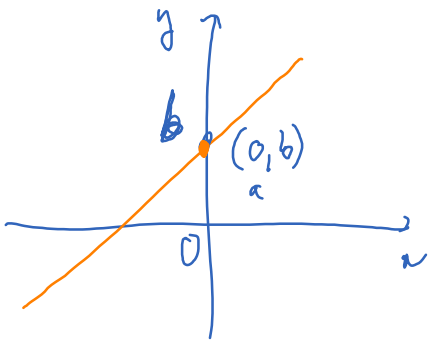
slope = 2

$y-1=2(x-1)$

$x=50$

$y-1=2(50-1) \rightarrow y=\cdots$

slope - intercept formula



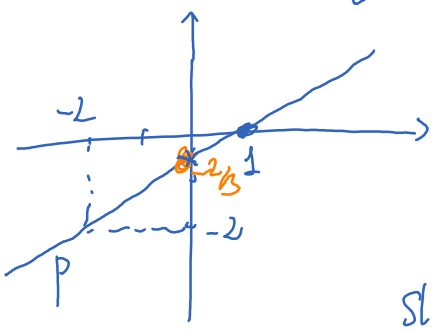
slope $= m$

y - intercept $= b$

$y-b=m(x-0)=mx$

$\boxed{y=mx+b}$ slope - intercept formula

Ex Write formula of function if whose graph is a straight line passing $P(-2,-2)$, $Q(1,0)$



$\text{slope} = \frac{\text{out. change}}{\text{in. change}} = \frac{0-(-2)}{1-(-2)} = \boxed{\frac{2}{3}}$

$\boxed{P(-2,-2)}$

a b

Slope-point form:

$y-(-2)=\frac{2}{3}(x-(-2))$

$\boxed{y+2=\frac{2}{3}(x+2)} \rightsquigarrow \boxed{y=\frac{2}{3}x-\frac{2}{3}}$ slope intercept