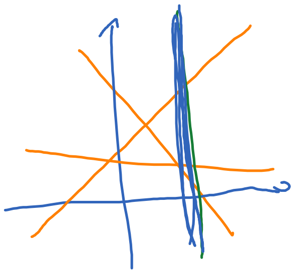
# Lecture 6

Monday, November 7, 2022 2:43 PM

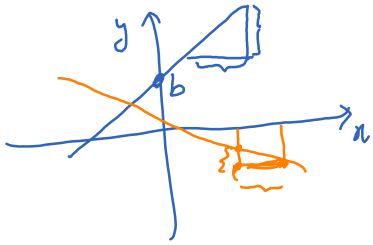
Linear function



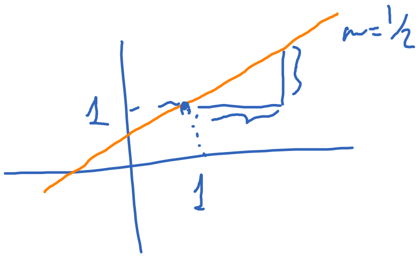
Except for the vertical straight lines, every str. line is a graph of a linear function.

$y = f(x) = mx + b$ : slope-intercept

$m$: slope, $b$: y-intercept




$y - b = m(x - a)$ : slope-point

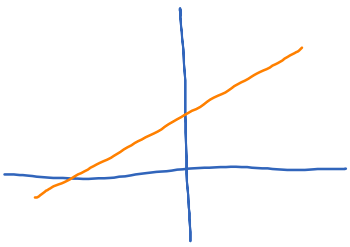



$$y - 1 = \frac{1}{2}(x-1) = \frac{1}{2}x - \frac{1}{2}$$

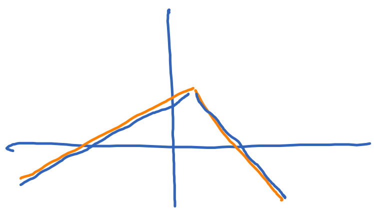
Add 1:

$$\underbrace{y - 1 + 1}_{} = \underbrace{\frac{1}{2}x - \frac{1}{2} + 1}_{}$$

$$y = \frac{1}{2}x + \frac{1}{2}$$ slope-intercept



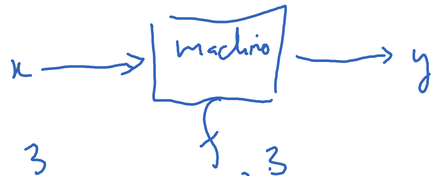
linear

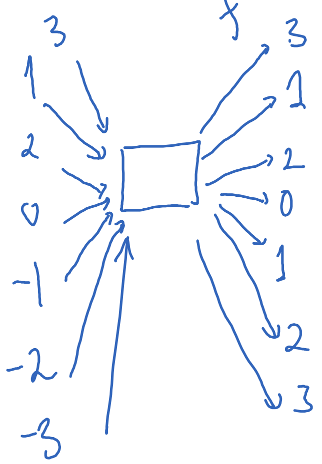


piecewise linear function

## Absolute-value function

$$y = f(x)$$




If $x > 0$: $f(x) = x$

If $x = 0$: $f(x) = x$

If $x < 0$: $f(x) = -x$

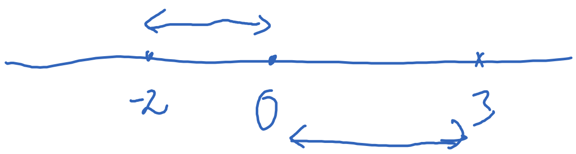
$$f(-2) = -(-2) = 2$$

$$-(-2) = 2$$

$$\boxed{f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}}$$

Absolute-value function

$f(x) = |x|$ : absolute value of $x$




$|x| =$ how far $x$ is from 0.

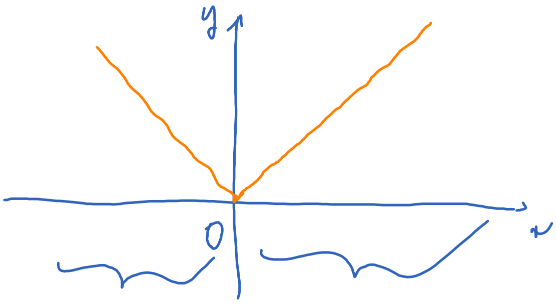
$|3| = 3$

$|-2| = 2$

$|0| = 0$

$$|x| = \begin{cases} \boxed{x} & \text{if } x \geq 0 \\ \boxed{-x} & \text{if } x < 0 \end{cases}$$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$




$f(x) = |x| = x$

slope $= 1$

$f(x) = -x$

slope $= -1$

$\sqrt{x^2} = x$ ? wrong

$\sqrt{x^2} = |x|$ ✓

$(-3)^2 = (-3)(-3) = 9$

$\sqrt{(-3)^2} = \sqrt{9} = 3$

Ex

$f(x) = |x+2| - |2x-2|$

$f(-3) = |-3+2| - |2(-3)-2| = |-1| - |-8| = 1 - 8 = -7$

$f(2) = |2+2| - |2(2)-2| = |4| - |2| = 4 - 2 = 2$

How to graph function $f$?

Strategy: remove vertical bars

$$f(x) = |x+2| - |2x-2|$$

$x+2 > 0$

$< 0$

$x+2$ vs $0$

$\boxed{x \text{ vs } -2}$

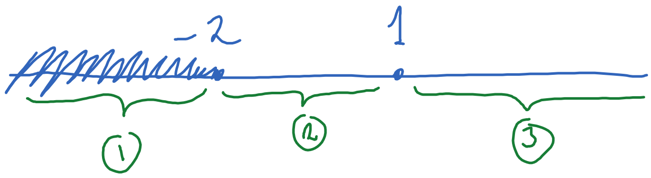
$2x-2 > 0$

$< 0$

$2x-2$ vs $0$

$2x$ vs $2$

$\boxed{x \text{ vs } 1}$




* When $x$ is in 1 : $(x < -2)$

$x < -2$

$x+2 < -2+2 = 0$

$2x-2$

$2(-3)-2 = -6-2 < 0$

$$f(x) = \underbrace{|x+2|}_{-(x+2)} - \underbrace{|2x-2|}_{-(2x-2)}$$

$$= -(x+2) + (2x-2)$$

$$= -x-2+2x-2$$

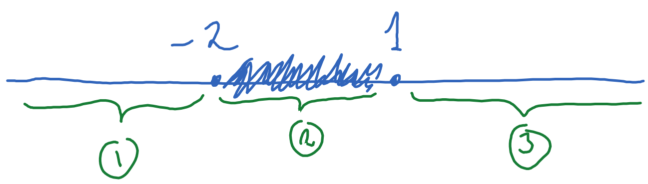
$$f(x) = x-4$$

* When $x$ is in 2 : $-2 < x < 1$

$$f(x) = \overbrace{|x+2|}^{x+2} - \overbrace{|2x-2|}^{-(2x-2)}$$

$x+2 > 0$

$2x-2 < 0$




$$f(x) = x+2 - -(2x-2)$$

$$= x + \cancel{2} + 2x - \cancel{2}$$

$$f(x) = 3x$$

* When $x > 1$: $f(x) = -x + 4$.

$$f(x) = \begin{cases} x - 4 & \text{if } x < -2 \\ 3x & \text{if } -2 < x < 1 \\ -x + 4 & \text{if } x > 1 \end{cases}$$

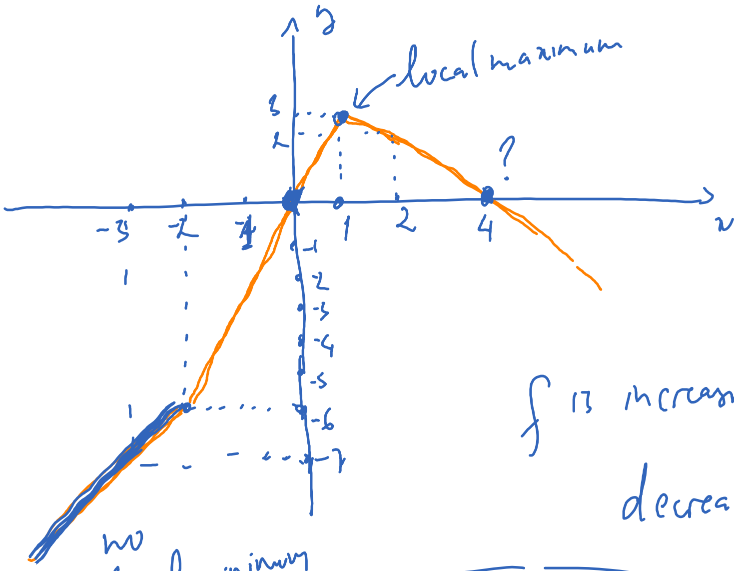
$f(x) = 0$

$x - 4 = 0$

$x = 4$

$x = 0$

$-x + 4 = 0$, $x = 4$




$f$ is increasing on $(-\infty, 1]$

decreasing on $[1, \infty)$

x, y intercepts of $f$?

y-intercept: plug $x = 0$

$f(0) = |0+2| - |2(0) - 2| = |2| - |-2| = 0$

$x = 0, y = 0$

y-int. is the origin

x-intercept. $f(x) = 0$ solve for $x$

$|x+2| - |2x-2| = 0$

x-intercepts are $(0,0)$ and $(4,0)$

$$|x+2| - |2x-2| = 0$$

$$\underbrace{|x+2|}_{\pm(x+2)} = \underbrace{|2x-2|}_{\pm(2x-2)}$$

① $x+2 = 2x-2 \rightsquigarrow x = 4$

② $x+2 = -(2x-2) \rightsquigarrow x = 0$