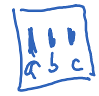
Lecture 7

Tuesday, November 15, 2022 9:15 PM

Quadratic function: Domain is $(-\infty, \infty)$

$$f(x) = ax^2 + bx + c \qquad (a \neq 0)$$

a, b, c are given numbers.

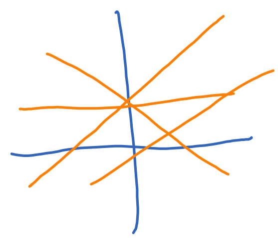


Ex

$$f(x) = x^2 = \underset{a}{1}\,x^2 + \underset{b}{0}\,x + \underset{c}{0}$$

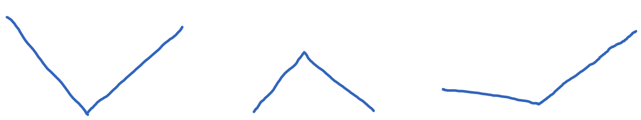
$$f(x) = x - x^2 = \underset{a}{-1}\,x^2 + \underset{b}{1}\,x + \underset{c}{0}$$

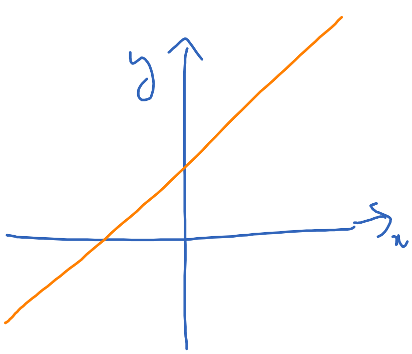
Linear function



$$f(x) = mx + b$$

given number





$$f(x) = ax^2 + bx + c$$

a: opening of the parabola

$a > 0$: parabola facing upward

$a < 0$: " " downward

a : opening of parabola
facing upward / downward

b : not so clear

c : y-intercept

$f(0) = a\cancel{0}^2 + b\cancel{0} + c = c$ is the y-intercept

$f(x) = ax^2 + bx + c$: general form of a quadratic function

$f(x) = a(x-h)^2 + k$: standard form of quad. function.

Ex

$$f(x) = x^2 = 1x^2 + 0x + 0$$
$$= \underset{a}{1}(x - \underset{h}{0})^2 + \underset{k}{0}$$

Ex

$$f(x) = \underset{a=-1}{-}(x - \underset{h}{1})^2 + \underset{k}{2}$$ standard form

general form:

$$f(x) = -(x-1)(x-1) + 2 = -(x^2 - x - x + 1) + 2$$
$$= -x^2 + x + x - 1 + 2 = \underset{a=-1}{-x^2} + \underset{b=2}{2x} + \underset{c=1}{1}$$

general form to standard form:

Ex $f(x) = 2x^2 - 4x + 9$ $[a=2, b=-4, c=9]$

$= 2(x-h)^2 + k$ h, k?

?

"completing a square"

$(x-h)^2 = (x-h)(x-h) = x^2 - 2hx + h^2$

$f(x) = \underline{2x^2 - 4x + 9}$

$= 2\left(x^2 - 2x + \frac{9}{2}\right)$

$= 2\left[(x-1)^2 - 1 + \frac{9}{2}\right] = 2\left[(x-1)^2 + \frac{7}{2}\right]$

$= 2(x-1)^2 + 2 \times \frac{7}{2}$

$= \boxed{2(x-1)^2 + 7}$

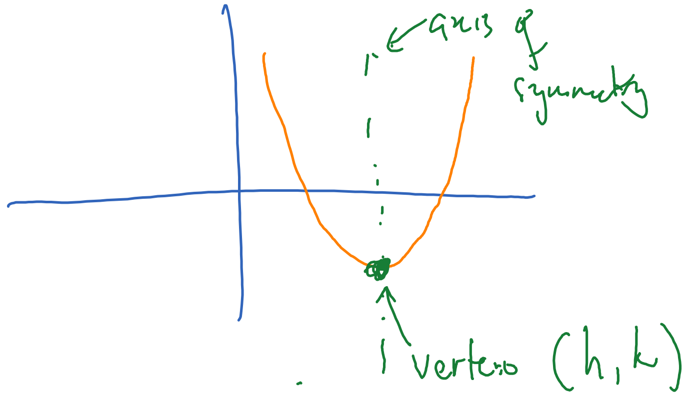
Standard form: $f(x) = a(x-h)^2 + k$

free constant

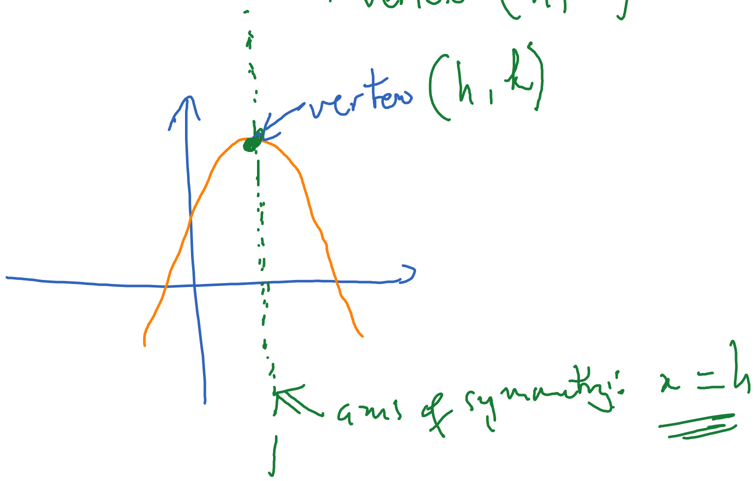
a: openning of the parabola.

h: indicate the position of the parabola in the horizontal direction.

k: indicate the position of the parabola in vertical direction



$$\boxed{f(x) = a(x-h)^2 + k}$$



Ex

$$f(x) = x^2 - 4x + 5$$

Find vertex and axis of symmetry.

$$x^2 - 4x + 5 = \overbrace{x^2 - 4x + 4} + 1 = \boxed{(x-2)^2 + 1}$$

$$(x-h)^2 = x^2 - 2hx + h^2$$

$$h = 2$$

$$(x-2)^2 = \underline{x^2 - 4x + 4}$$

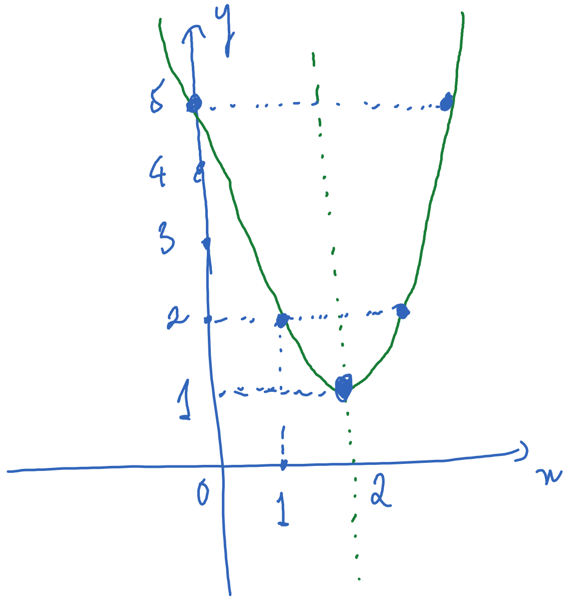
complete square

"incomplete square"

$\underline{a = 1}$, $h = 2$, $k = 1$

$f(x) = (x-2)^2 + 1$ $\quad f(1) = (1-2)^2 + 1 = 2$

$a=1, \quad h=2, \quad k=1$



$x=2$ is the axis of sym.

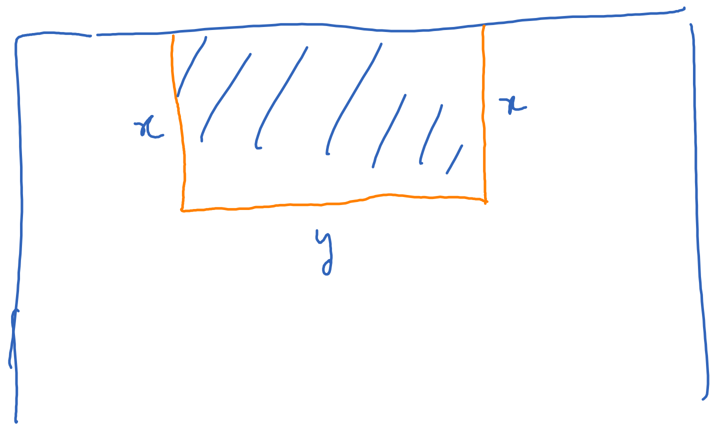
$(2,1)$ is coord. of vertex.

when $x=0$: $f(0) = (0-2)^2 + 1$
$= 5$

To graph a quad. function:

- Find vertex (h,k)
- Draw axis of symmetry: $x=h$
- Find one more point on the graph.
- Reflect through the axis of symmetry

Ex Home Depot 100 ft of wire fence.



$x+x+y=100$

x ? maximum enclosed area

Area $= xy$

$$2x+y=100 \rightsquigarrow y = 100-2x$$

$$\text{Area} = xy = x(100-2x) = 100x - 2x^2$$

$$= -2x^2 + 100x + 0$$

quadratic function!

Find x : $-2x^2+100x$ max

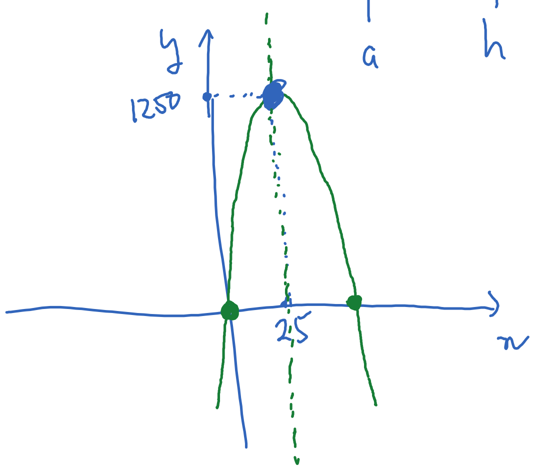
Standard: $-2(x^2-50x) = -2(\underbrace{x^2 - 2\times 25 x + 25^2} - 25^2)$

$$\boxed{x^2 - 2hx + h^2 = (x-h)^2}$$

$$= -2\left((x-25)^2 - 625\right)$$

$$-2x^2 + 100x = -2(x-25)^2 + 1250$$

$$\boxed{-2x^2+100x} = \underset{a}{-2}(x-\underset{h}{25})^2+\underset{k}{1250}$$



$x=0,\ y=0$

$\max = \underline{1250} = \underline{\text{max area}}$

$\underline{x=25}$