MATH 111A, MIDTERM, FALL 2022

INSTRUCTOR: TUAN PHAM

Name

Instructions:

- This is a closed-book exam, 90 minutes long.
- A non-graphing calculator is allowed. Scratch paper is allowed.
- For Problems 1-11, fill in the bubbles on this front page. To each problem, only one answer is correct.
- For Problems 12 and 13, make sure to show all necessary steps. Mysterious answers will receive little or no credit.
- Do not discuss the exam with anyone during Nov 3-8.

1. A B C **D**
2. A B **C** D
3. A **B** C D
4. A **B** C D
5. **A** B C D
6. A B C **D**
7. A B **C** D
8. **A** B C D
9. A B **C** D
10. **A** B C D
11. A B C **D**

Problem	Possible points	Earned points
1-11	22	
12	5	
13	5	
Total	32	

Problem 1. (2 points) Which of the following interval notations describe the set

$$\{x|\, x < 5 \text{ or } x \geq 2\}$$

A. $[2, 5)$

B. $(-\infty, 5) \cup [2, \infty)$

C. $(-\infty, \infty)$

D. Both B and C

Problem 2. (2 points) The points $A(1, -2)$ and $B(-2, -3)$ lie in the quadrants

A. I and II

B. II and III

C. IV and III

D. I and III

Problem 3. (2 points) Consider three points $A(1, 1)$, $B(-1, 2)$, $C(-1, 0)$. Which side of the triangle ABC is the shortest side?

A. AB

B. BC

C. CA

D. All sides have the same length.

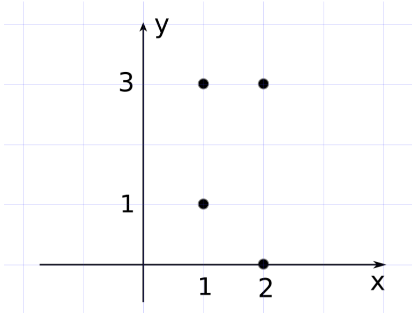
Problem 4. (2 points) The graph of the relation $\{(x, 1)|\, x \in \mathbb{R}\}$ is

A. a line parallel to the y-axis and intersecting the x-axis at $x = 1$.

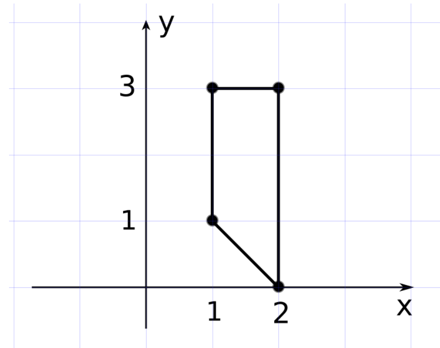
B. a line parallel to the x-axis and intersecting the y-axis at $y = 1$.

C. the x-axis.

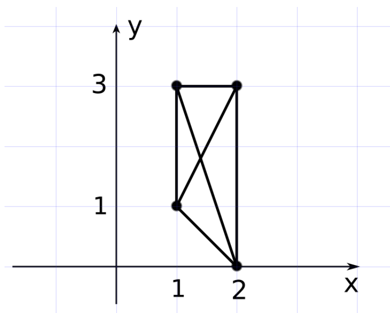
D. the y-axis.



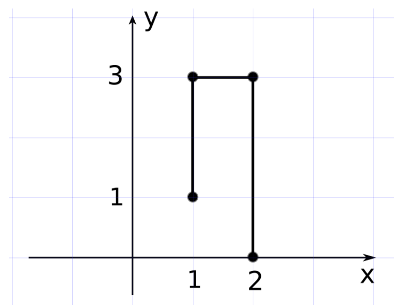
(A)



(B)



(C)



(D)

Problem 5. (2 points) Determine from the above pictures the correct graph of the relation

$$R = \{(1, 1), (1, 3), (2, 3), (2, 0)\}.$$

A. Graph (A)
B. Graph (B)
C. Graph (C)
D. Graph (D)

Problem 6. (2 points) The function $f(x) = -x^2 + x$ is

A. an odd function.

B. an even function.

C. both even and odd.

D. neither even nor odd.

Problem 7. (2 points) Determine the domain of the function

$$f(x) = \frac{\sqrt{x-1}}{x-2}.$$

A. $(-\infty, 1)$

B. $[1, \infty)$

C. $[1, 2) \cup (2, \infty)$

D. $(1, 2) \cup (2, \infty)$

Problem 8. (2 points) The graph of function $g(x) = \frac{1}{x+1}$ can be obtained from the graph of function $f(x) = \frac{1}{x}$ by

A. shifting the graph of $f(x)$ to the left 1 unit.

B. shifting the graph of $f(x)$ to the right 1 unit.

C. shifting the graph of $f(x)$ up 1 unit.

D. shifting the graph of $f(x)$ down 1 unit.

Problem 9. (2 points) A function f takes a real number x and performs the following three steps in the order given: (1) square; (2) subtract 1; (3) make the quantity the denominator of a fraction with numerator 2. Determine the correct expression of $f(x)$.

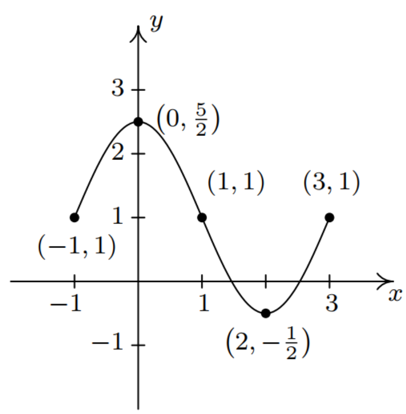
A. $\frac{\sqrt{x}-1}{2}$

B. $\frac{2}{\sqrt{x}-1}$

C. $\frac{2}{x^2-1}$

D. $\frac{x^2-1}{2}$

Problem 10. (2 points) The graph of a function f is given in the picture below. On what interval is



f increasing?

A. $[-1, 0]$

B. $[-1, 1]$

C. $[0, 2]$

D. $[1, 3]$

Problem 11. (2 points) Let $f(x) = \frac{x^2-x}{x+1}$. Find x such that $f(x) = 0$.

A. 0

B. 1

C. -1

D. Both A and B

Problem 12. (5 points) Let $f(x) = -x^2$. Simplify

$$\frac{f(x+h) - f(x)}{h}$$

Make sure to show all your computation steps.

$$f(x+h) = -(x+h)^2 = -x^2 - 2xh - h^2$$

$$f(x) = -x^2$$

$$f(x+h) - f(x) = -2xh - h^2 = h(-2x - h)$$

$$\frac{f(x+h) - f(x)}{h} = -2x - h$$

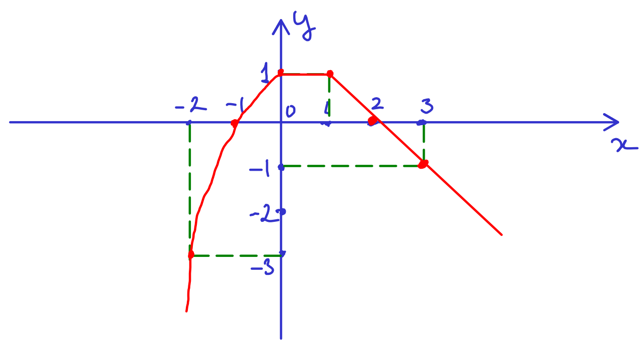
Problem 13. (5 points) Let f be a function defined piecewise as follows.

$$f(x) = \begin{cases} 1 - x^2 & \text{if} \quad x < 0, \\ 1 & \text{if} \quad 0 \le x \le 1, \\ 2 - x & \text{if} \quad x > 1. \end{cases}$$

(a) Make a table of at least 5 values of x in the interval $[-2, 3]$ and corresponding values of $y = f(x)$. Then sketch the function.

(b) Find the x-intercepts and y-intercepts of the graph. (You need to write an equation and solve it. Don't just rely on the graph.)

x	$f(x)$
-2	-3
-1	0
0	1
1	1
2	0
3	-1



To find the y-intercept, we only need to compute $f(0)$.

$f(0) = 1$ (based on the definition of f)

To find the x-intercepts, we set $f(x) = 0$ and solve for x.

Such x must be either less than 0 or greater than 1 because f is equal to 1 when x is between 0 and 1.

- If $x < 0$: $f(x) = 1 - x^2 = 0$ gives $x^2 = 1$, i.e. $x = \pm 1$.

  Only -1 is chosen because it is less than 0.

- If $x > 1$: $f(x) = 2 - x = 0$ gives $x = 2$.

Therefore, there are two x-intercepts: $x = -1$ and $x = 2$.