Graphing functions using Calculus

Friday, December 2, 2022 11:03 AM

Things to consider when graphing a function $f(x)$

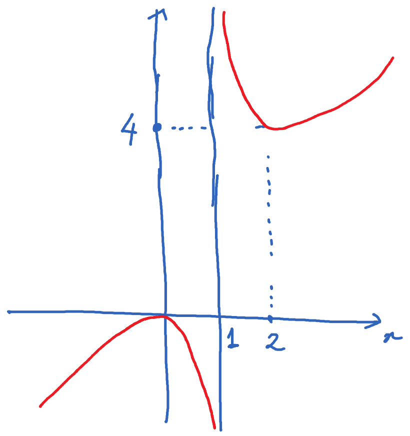
- Domain
- Symmetry (even/odd)
- x,y- intercepts
- local min/max
- intervals of increasing/decreasing
- assymptotes
- concavity
- inflection points

Ex graph the function $f(x) = \dfrac{x^2}{x-1}$ $\begin{matrix} \to 1 \\ \to 0^- \end{matrix}$ $x \to 1^-$

Domain $= \mathbb{R} \setminus \{1\}$ x,y-intercepts : $(0,0)$

$$f'(x) = \frac{2x(x-1) - x^2}{(x-1)^2} = \frac{x^2-2x}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2}$$

Critical points are $x = 0$ and $x = 2$.



"Fluctuation" chart:

x		0		1		2	
f'	$+$	0	$-$	‖	$-$	0	$+$
f	$-\infty \nearrow$	0	$\searrow -\infty$	‖	$\infty \searrow$	4	$\nearrow \infty$

$$f''(x) = \left(\frac{x^2-2x}{(x-1)^2}\right)' = \frac{(2x-2)(x-1)^2 - (x^2-2x)2(x-1)}{(x-1)^4} = \frac{2}{(x-1)^3} \neq 0$$

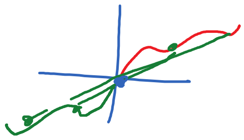
No inflection point. When $x < 1$, graph is concave downward
$x > 1$, " " upward

Ex graph function $f(x) = \dfrac{x}{x^2-9}$ $\qquad \lim\limits_{x\to -3^-} \dfrac{\overset{<0}{x}}{\underset{>0}{x^2-9}} = -\infty$

- Domain $\mathbb{R}\setminus\{-3,3\}$

- Symmetry

$$f(-x) = \frac{-x}{(-x)^2-9} = \frac{-x}{x^2+9} = -f(x)$$



f is an odd function

graph of f is symmetric w.r.t the origin

- x,y-intercepts.

x-intecept $\quad f(x)=0 \longrightarrow x=0$

y-incept. $\quad$ set $x=0$, $y=f(0)=0$

$\Big\}$ $(0,0)$ is an x-int & y-int

- assymptotes

vertical assymptotes $\quad x=-3, \; x=3$

horizontal assymptotes: $\quad \lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{x}{x^2-9} = 0$

$\qquad\qquad\qquad\qquad\quad\;\; \lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \dfrac{x}{x^2-9} = 0$

$\Big\}$ $y=0$ is the only horizontal assymptote

- local min/max, interval of increasing/decreasing

$$f'(x) \overset{\text{quotient rule}}{=\!=\!=} -\frac{\overset{>0}{x^2+9}}{\underset{>0}{(x^2-9)^2}} < 0$$

, no critical number.

f is a decreasing function

Fluctuation chart

x		-3		3	
f'	$-$	‖	$-$	‖	$-$
f	$0 \searrow -\infty$	‖	$\infty \searrow -\infty$	‖	$\infty \searrow 0$

$$f''(x) = \left(-\frac{x^2+9}{(x^2-9)^2} \right)' \underset{\text{quotient rule}}{=} \frac{2x(x^2+27)}{(x^2-9)^3}$$

$f''(x) = 0$ when $x = 0$ ← inflection point

x		-3		0		3	
f'	$-$		$-$	$-$	$-$		$-$
f''	$-$		$+$	0	$-$		$+$
f	$0 \searrow -\infty$		$\infty \searrow$	0	$\searrow -\infty$		$\infty \searrow 0$