Maple Lab 4

In this lab, you will learn how to use Maple to

- plot a curve given by an equation,
- compute implicit differentiation,
- find local/global minimum/maximum of a function.

1 Practice

Plotting an equation and performing implicit differentiation:

If a curve is given by an equation

$$y = f(x), \tag{1}$$

one can plot it using the command `plot(f(x),x=a..b)`. However, not every curve is a graph of a function. The unit circle, for example, is not the graph of any function because it does not pass the Vertical Line Test. Nevertheless, it has a nice equation: $x^2 + y^2 = 1$. More generally, any equation involving x and y, not necessarily of the form (1), describes a curve. For example, the equation $x^2 - xy = y^3$ describes a curve which consists of all the points (x, y) satisfying that equation.

To plot such a curve, one can use the command `implicitplot` in the package `plots` of Maple. This package consists of a number of functionalities specialized for plotting. To call it, you enter the command

```
with(plots);
```

Now that the package is called, you can use the command

```
implicitplot(equation,x=a..b,y=c..d);
```

to plot the curve. Here a, b, c, d specify the window of x and y we want to observe. Maple will search in these ranges for pairs (x, y) that satisfy the equation. For example, to graph the equation $x^2 - xy = y^3$ in the window $-2 \leq x \leq 2$, $-2 \leq y \leq 2$, you enter the command

```
implicitplot(x^2-x*y=y^3,x=-2..2,y=-2..2);
```

The result is Figure 1.

The curve is a quite rough. To improve the smoothness of the curve, you need to specify the value of `gridrefine`, the level of "refinement". The default value of `gridrefine` is 0. The larger `gridrefine` is, the smoother the curve. For example, try

```
implicitplot(x^2-x*y=y^3,x=-2..2,y=-2..2,gridrefine=3);
```

To perform implicit differentiation on an equation involving x and y is to find y' in terms of x and y. For example, consider the equation $x^2 - xy + y^2 = 1$. To find y', you can use the command `implicitdiff` with the following syntax:

```
implicitdiff(x^2-x*y+y^2=1,y,x);
```

The result is y' (when y is viewed as a function of x). If you switch the order of x and y:

```
implicitdiff(x^2-x*y+y^2=1,x,y);
```

the result is x', which is the derivative of x with respect to y when x is viewed as a function of y.

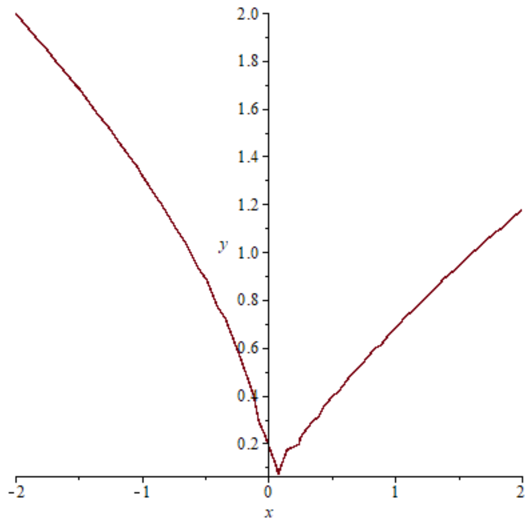

Figure 1

Optimization problems:

If a function f is differentiable on an interval (a, b) and has a peak (local maximum) or a valley (local minimum) at $c \in (a, b)$, then $f'(c) = 0$. This is known as the *Fermat's theorem*. A point x such that $f'(x) = 0$ is called a *critical value* of f.

Let us consider an example where $f(x) = 3x^4 - 4x^3 - 12x^2 + 1$. To find all the critical points of f is to find all the roots of the equation $f'(x) = 0$. One can do so on Maple thanks to the command `solve`:

```
f:=x->3x^4-4x^3-12x^2+1;
solve(f'(x)=0,x);
```

The result is $x = -1, 0, 2$. Note that the command `solve` aims to give exact roots of an equation. Sometimes, the exact roots do not have a nice form, or it may take Maple too long to compute. In that case, you can replace `solve` with `fsolve`, which will give you an approximate numerical value of the roots.

To find the absolute minimum or absolute maximum of f on the closed interval $[-2, 3]$, we first compute all the critical points of f, i.e. the roots of $f'(x) = 0$, between -2 and 3. Then we compare the values of f at these points with $f(-2)$ and $f(3)$. Specifically, you compare five numbers $f(-2)$, $f(-1)$, $f(0)$, $f(2)$, $f(3)$. The largest of them is the maximum. The smallest of them is the minimum.

You can also use the commands `minimize` and `maximize` on Maple:

```
minimize(3*x^4-4*x^3-12*x^2+1,x=-2..3);
maximize(3*x^4-4*x^3-12*x^2+1,x=-2..3);
```

which give -31 and 33. To know the value(s) of x where $f(x)$ is maximum or minimum, you add the option `location` into the above commands. For example,

```
minimize(3*x^4-4*x^3-12*x^2+1,x=-2..3,location);
maximize(3*x^4-4*x^3-12*x^2+1,x=-2..3,location);
```

2 To turn in

1. Consider a curve with the equation

$$2y^3 + y^2 - y^5 = x^4 - 2x^3 + x^2$$

   (a) Use Maple to plot the curve.

   (b) At what points does this curve have horizontal tangent lines? Find the coordinates of these points.
   (Hint: find y' using `implicitdiff` and then set $y' = 0$ to solve for x.)

   (c) At what points does this curve have vertical tangent lines? Find the coordinates of these points.

2. Let $f(t) = 2\cos t + \sin(2t)$.

   (a) Graph the function f on the interval $[0, 2\pi]$.

   (b) Find all the critical points of f. For which value of t does f attain local maximum or local minimum?

   (c) Find the absolute maximum and absolute minimum of f on the interval $[0, 2\pi]$. At what value(s) of t are the absolute minimum and maximum attained?