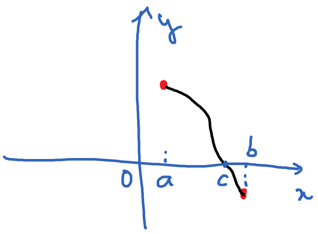
Lecture 10

Monday, October 17, 2022 8:34 AM

* Question

Intermediate value theorem



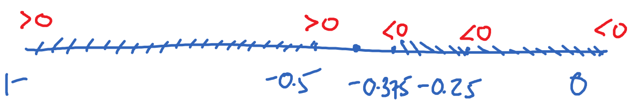
If f is continuous on $[a,b]$ and $f(a)$, $f(b)$ have different signs then

$f(c)=0$ for some $c\in(a,b)$.

Note the theorem doesn't tell us what c is. But we can find an approximate value of c up to any prescribed precision

Ex

Solve for $x\in(-1,0)$ such that $\underbrace{x^3-3x-1}_{f(x)}=0$ with precision 0.01.



Bisection method (a type of binary search algorithm)

The interval to search for the root get smaller each time

[worksheet problem 1]

* Limit at infinity

Comparison of polynomials

$$\lim_{x\to\infty}\frac{x^2}{x^2+x+2}=\lim_{x\to\infty}\frac{\frac{x^2}{x^2}}{\frac{x^2+x+2}{x^2}}=\lim_{x\to\infty}\frac{1}{1+\frac{1}{x}+\frac{2}{x^2}}=\frac{1}{1+0+0}=1$$

dominant term

insignificant terms

$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots$

$Q(x) = b_m x^m + b_{m-1} x^{m-1} + \cdots$

$$\lim_{x \to \infty} \frac{P(x)}{Q(x)} = \begin{cases} \frac{a_n}{b_m} & \text{if } m = n \\ 0 & \text{if } m > n \\ \infty \ (\text{or } -\infty) & \text{if } m < n \end{cases}$$

[do the remaining worksheet problems next time]