Lecture 13

Thursday, October 20, 2022 11:47 PM

* Questions?

Derivative of a function f at t is $f'(t) = \lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$

Applications of derivatives:

- speed
- rate of change
- slope

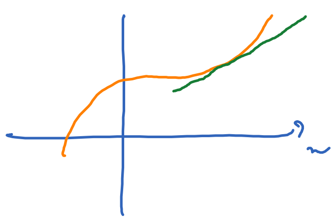
$T' = -k(T - 70)$

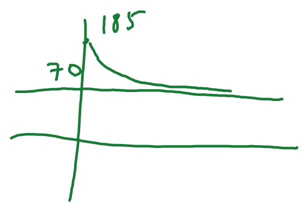
$T' = 70k\ Tk \longrightarrow T' + \alpha T = 70\alpha$

$T(t) = Ce^{-\alpha t} + 70$

$T = 115e^{-\alpha t} + 70$

Ex



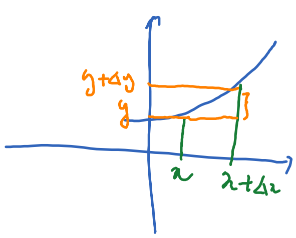


$T(0) = 185$

$T(60) = 115e^{-\cancel{60}} + 70$

$115e^{-\frac{t}{70}} + 70$

$T(0) = 185$



$T = T(t)$

T' = rate of change of temperature

$$\frac{f(t+\Delta t) - f(t)}{\Delta t} = \text{average rate of change of } f \text{ at } t.$$

$$\lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t} = \text{instantaneous rate of change of } f \text{ at } t$$

Ex temperature of turkey at initial time = 185

Room temperature = 70

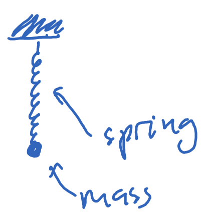
After $\underbrace{1 \text{ hour}}_{60 \text{ minute}}$, temperature of turkey = 75

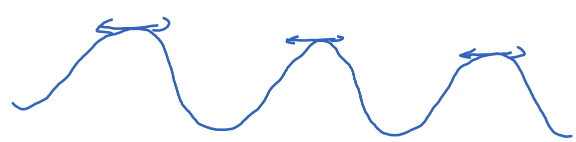
$T = T'(t)$

average rate of change of T is $\frac{75-185}{60} = \frac{110}{60} = \frac{11}{6} \approx -1.83$

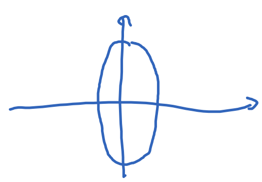
Newton's law of cooling $T' = \alpha(T_0 - T)$

ambient temperature





Slope



$2x^2 + y^2 = 6$ $\qquad \frac{x^2}{3} + \frac{y^2}{6} = 1$

$x=1, y=2$

$y = \sqrt{6-2x^2}$

$$\frac{\sqrt{6-2(1+h)^2} - 2}{h} = \frac{2-2(1+h)^2}{h\left(\sqrt{6-2(1+h)^2}+2\right)}$$

$$= \frac{-4h-4h^2}{h\left(\sqrt{6-2(1+h)^2}+2\right)}$$