Lecture 17

Friday, October 28, 2022 8:42 AM

* Questions ....

Notations of derivatives: $y = f(x)$

f', $\frac{dy}{dx}$, $\frac{df}{dx}$, y', $\frac{d}{dx}f$: equivalent notations

f'', $\frac{d^2y}{dx^2}$, $\frac{d^2f}{dx^2}$, y'', $\frac{d^2}{dx^2}f$ equivalent notations

Product rule: $y = f(x)$, $z = g(x)$

$$\frac{d(yz)}{dx} = ?$$

$$\frac{(y+\Delta y)(z+\Delta z) - yz}{\Delta x} = \frac{y\Delta z + z\Delta y}{\Delta x} = y\frac{\Delta z}{\Delta x} + z\frac{\Delta y}{\Delta x} = fg' + gf'.$$

($y \to f$, $\frac{\Delta z}{\Delta x} \to g'$; $z \to g$, $\frac{\Delta y}{\Delta x} \to f'$)

Product rule $(fg)' = f'g + fg'$

Ex. $(x^2)' = (xx)' = x'x + xx' = x + x = 2x$

$(x^3)' = (xxx)' = x'xx + xx'x + xxx' = x^2 + x^2 + x^2 = 3x^2$

$(x^4)' = (x^2x^2)' = (x^2)'x^2 + x^2(x^2)' = 2xx^2 + x^2 2x = 4x^3$

power rule. $(x^n)' = nx^{n-1}$

Work on the worksheet.

In Calc 3.

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

By the sum rule, scale rule, and power rule,

$$(\sin x)' = 1 - \frac{3x^2}{3!} + \frac{5x^4}{5!} - \frac{7x^6}{7!} + \cdots = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$= \cos x$$

Similarly,

$$(\cos x)' = -\sin x$$