

# Lecture 18

Friday, November 4, 2022 8:44 AM

\* Question

Differentiation rules :

- Sum / subtraction rule

- scale rule

- product rule

- quotient rule

- power rule

- **chain rule** - to differentiate a composite function

↑  
this is the rule we'll use most often

$$f(g(x)) = ?$$

Ex  $(\sin(x^2))' = ?$

$$[(\sin x)^2]' = ?$$

....

Analysis

$$y = g(x)$$

$$z = f(y)$$

Imagine:

$x$  is time

$y$  is distance travelled

$z$  is gas consumed

$$z = f(y) = f(g(x))$$

$\Delta x$ : change in  $x$   
 $\Delta y$ : change in  $y$  caused by change in  $x$   
 $\Delta z$ : change in  $z$  caused by change in  $y$

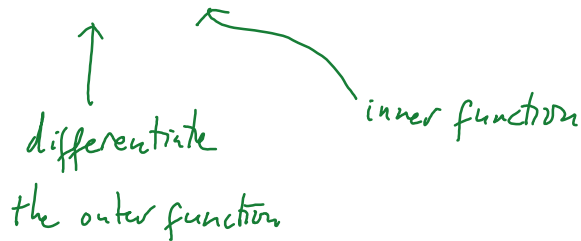
} Domino's effect

$$\frac{\Delta z}{\Delta x} \xrightarrow{\Delta x \rightarrow 0} z'(x) = [f(g(x))]'$$

$$\frac{\Delta z}{\Delta x} = \frac{\Delta z}{\Delta y} \frac{\Delta y}{\Delta x} \xrightarrow{\Delta x \rightarrow 0} z'(y)y'(x) = f'(y)g'(x) = f'(g(x))g'(x).$$

Therefore,

$$[f(g(x))]' = f'(g(x))g'(x)$$


  
 differentiate the outer function

Peeling an onion: one layer at a time, starting from the outermost layer.

$$[f(g(h(x)))]' = f'(g(h(x)))g'(h(x))h'(x)$$


  
 multiply them

This is similar, but not the same as product rule where we add instead

of multiply;  $(fgh)' = f'gh + fg'h + fgh'$

$$\underline{\underline{Ex}} \quad [\sin(x^2)]' \quad g(x) = x^2$$

$$\parallel \quad f(x) = \sin x$$

$$\underbrace{f'(g(x))}_{\cos(x^2)} \underbrace{g'(x)}_{2x} = 2x \cos 2x$$

$$\underline{\underline{Ex}} \quad [(\sin x)^2]' = g'(f(x))f'(x) = 2f(x)f'(x) = 2 \sin x \cos x.$$

$$\underline{\underline{Ex}} \quad [\sin(\sin(x^2))]' = \cos(\sin(x^2)) \cos(x^2) 2x$$

When the chain rule becomes our muscle memory (like driving), we don't need to write  $f, g, \dots$  anymore.