Lecture 18

Friday, November 4, 2022 8:44 AM

* Question

Differentiation rules :

- Sum / subtraction rule
- scale rule
- product rule
- quotient rule
- power rule
- chain rule . to differentiate a composite function

this is the rule we'll use most often

$f(g(x)) = ?$

Ex $\left(\sin(x^2)\right)' = ?$

$\left[(\sin x)^2\right]' = ?$

....

Analysis

$y = g(x)$

$z = f(y)$

Imagine : x is time

y is distance travelled

z is gas consumed

$z = f(y) = f(g(x))$

Δx : change in x

Δy : change in y caused by change in x } Domino's effect

Δz : change in z caused by change in y

$$\frac{\Delta z}{\Delta x} \xrightarrow{\Delta x \to 0} z'(x) = [f(g(x))]'$$

$$\frac{\Delta z}{\Delta x} = \frac{\Delta z}{\Delta y}\frac{\Delta y}{\Delta x} \xrightarrow{\Delta x \to 0} z'(y)\,y'(x) = f'(y)\,g'(x) = f'(g(x))\,g'(x).$$

Therefore,

$$[f(g(x))]' = f'(g(x))\,g'(x)$$

differentiate the outer function

inner function

Peeling an onion: one layer at a time, starting from the outermost layer.

$$[f(g(h(x)))]' = f'(g(h(x)))\,g'(h(x))\,h'(x)$$

multiply them

This is similar, but not the same as product rule where we add instead of multiply: $(fgh)' = f'gh + fg'h + fgh'$.

Ex $[\sin(x^2)]'$ $\qquad g(x) = x^2$

$\qquad f(x) = \sin x$

$$\| $$

$$\underbrace{f'(g(x))}_{\cos(x^2)}\ \underbrace{g'(x)}_{2x} \quad = 2x\cos 2x$$

Ex $[(\sin x)^2]' = g'(f(x))f'(x) = 2f(x)f'(x) = 2\sin x\cos x.$

Ex $[\sin(\sin(x^2))]' = \cos(\sin(x^2))\cos(x^2)\, 2x$

When the chain rule becomes our muscle memory (like driving), we don't need to write $f, g, \ldots$ anymore.