Lecture 19

Monday, November 7, 2022 10:32 AM

Questions .....

More examples on the chain rule.

Ex $y = y(x)$

$\frac{d}{dy}(y^4) = 4y^3$, $\frac{d}{dx}(y^4) = ?$

$x \xrightarrow{y} y(x) \xrightarrow{\wedge 4} y(x)^4$

$\frac{d}{dx}(y^4) = 4y(x)^3 y'(x) = 4y^3 y'$

Ex

$y = y(x)$

$\frac{d}{dx}[\sin\sqrt{y}] = ?$

$x \xrightarrow{y} y(x) \xrightarrow{\sqrt{\ }} \sqrt{y(x)} \xrightarrow{\sin} \sin\sqrt{y(x)}$

$\frac{d}{dx}[\sin\sqrt{y}] = \cos\sqrt{y(x)}\, \frac{1}{2\sqrt{y(x)}}\, y'(x) = \cos\sqrt{y} \cdot \frac{1}{2\sqrt{y}}\, y'.$

Tip:

Differentiate $\sin\sqrt{y}$ wrt y and then multiply by y'.

$\frac{d}{dx}[\sin\sqrt{y}] = \frac{d}{dy}[\sin\sqrt{y}]\, \frac{dy}{dx} = \cos\sqrt{y} \cdot \frac{1}{2\sqrt{y}} \cdot y'$

Work on the worksheet

Implicit differentiation: This is not a new rule of differentiation, only a name of an application of the chain rule.

$$y = f(x)$$

$$y' = f'(x): \text{ explicit differentiation}$$

Sometimes, y is not given as an explicit function of x, but given "implicitly" in an equation, for example

$$x^2 + y^2 = 1,$$

$$y\cos(xy) = 2,$$

$$\cdots$$

x and y can't be arbitrary, but have to coordinate very well with each other to satisfy the equation. Thus, y is implicitly a function of x.

Ex $\quad y\cos(xy) = 2$

Find y'

Diff. both sides wrt x:

$$\underbrace{\frac{d}{dx}[y\cos(xy)]}_{\text{product rule}} = \frac{d}{dx}2 = 0$$

$$= y'\cos(xy) + \underbrace{y\frac{d}{dx}[\cos(xy)]}$$

$$= y(-\sin(xy))(xy)' = -y\sin(xy)(x'y + xy')$$

Thus,

$$0 = y'\cos(xy) - y\sin(xy)(y + xy')$$

Thus,

$$0 = y'\cos(xy) - y\sin(xy)\,(y + xy')$$

$$= y'\left(\cos(xy) - xy\sin(xy)\right) - y^2\sin(xy)$$

$$y' = \frac{y^2\sin(xy)}{\cos(xy) - xy\sin(xy)}$$