Lecture 21

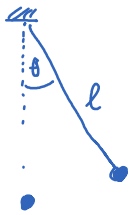
Thursday, November 10, 2022 2:32 PM

* Questions ....

* Work on worksheet practice using chain rule and other differentiation rules.

* Linear approximation.

Pendulum problem in physics.



The angle θ is a function of time. $\theta = \theta(t)$.

From the laws of physics, $\theta'' + \frac{g}{\ell}\sin\theta = 0$

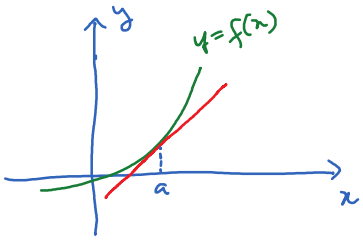
This is a differential equation to solve for θ as a function of t. It is difficult to solve because sine is a nonlinear function

If θ is small, then $\sin\theta \approx \theta$. We solve the equation

$$\theta'' + \frac{g}{\ell}\theta = 0$$

instead. It is much easier to solve (Math 321).

Approximating $\sin\theta$ with θ when θ is small is a linear approximation.



Consider a general function $f(x)$. We are interested in the value of $f(x)$ when x is near a.

Zooming in graph of f is very close to the tangent line of the graph at $x = a$.

Tangent line. $y - f(a) = f'(a)(x-a)$

or $y = f(a) + f'(a)(x-a)$.

Therefore, we get a "linear approximation"

$$\underbrace{f(x)}_{\text{nonlinear function}} \approx \underbrace{f(a) + f'(a)(x-a)}_{\text{linear function}}$$

This approximation is only good when x is close to a. The further x is from a, the worse the approximation.