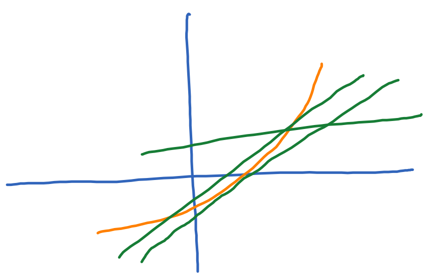
Lecture 22

Monday, November 14, 2022 8:46 AM

Questions ....

Linear approximation: this is to approximate a nonlinear function by a linear function.

Graphically, this is to approximate a curve by a straight line.



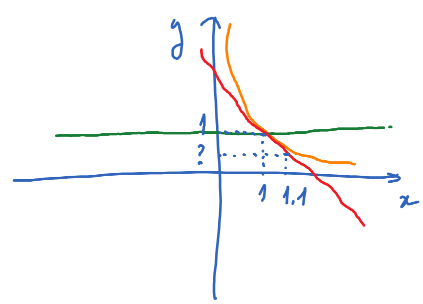
This is not linear regression that you learn in Statistics.

We want an approximate of $f(x)$ when x is close to a.

$$f(x) \approx f(a) + f'(a)(x-a)$$

Ex Approximate $\frac{1}{1.1}$

Naive: $\frac{1}{1.1} \approx \frac{1}{1} = 1$



$f(1.1) \approx ?$ where $f(x) = \frac{1}{x}$.

$f(1.1) \approx f(1) + f'(1)(1.1-1)$ $\qquad f'(x) = -\frac{1}{x^2}$

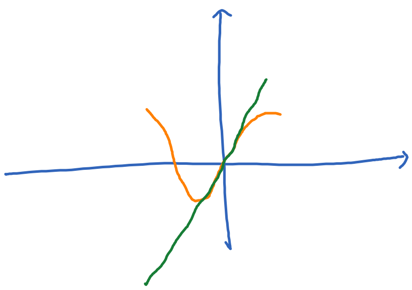
$= 1 + (-1)(0.1)$

$= 0.9$

Testing with calculator, we can see that 0.9 is a better approximation of $\frac{1}{1.1}$ than 1.

Ex. Approximate $\sin\theta$ when θ is small.

This is a problem that arises in Physics (the pendulum problem).



$$f(\theta) = \sin\theta$$

$$\approx f(0) + f'(0)(\theta - 0)$$

$$f(0) = \sin 0 = 0$$

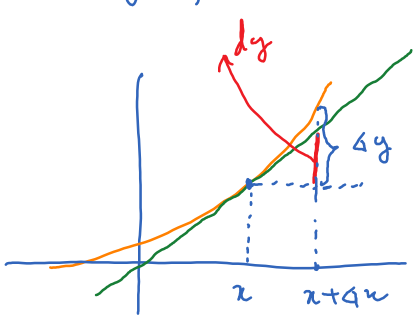
$$f'(\theta) = \cos\theta, \quad f'(0) = \cos 0 = 1$$

Thus, $f(\theta) \approx 0 + 1(\theta - 0) = \theta$.

$$\boxed{\sin\theta \approx \theta \text{ when } \theta \text{ is small.}}$$

Differential

$y = f(x)$



$$\Delta y = f(x + \Delta x) - f(x)$$

dy is the change in y after linear approximation.

$$\frac{dy}{dx} = f'(x) \text{ (slope of tangent line)}$$

$$dy = f'(x)\,dx$$

differential   derivative   any number

dx is usually treated as a symbol rather than a numerical value.