Lecture 25

Friday, November 18, 2022 1:13 AM

Questions ...

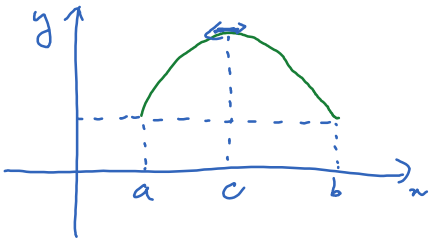
Continue to work on the worksheet (practice on optimization problems)

Fermat's lemma

If c is a local min/max of f and f is differentiable at c then $f'(c)=0$.

Rolle's thm

If f is continuous on $[a,b]$ and differentiable on (a,b) and $f(a)=f(b)$ then $\exists\, c\in(a,b)$: $f'(c)=0$.



Ex Show that the equation $\sin x = 2x$ has only one root.

It is clear that $x=0$ is a root.

Suppose that the equation has two different roots, say a and b.

Then $f(a)=f(b)=0$, where $f(x)=\sin x - 2x$.

By Rolle's thm, $\exists\, c\in(a,b)$ such that $f'(c)=0$.

However, $f'(c) = \cos c - 2 \leq 1 - 2 = -1 < 0$.

This is a contradiction. Therefore, there is only one root of $\sin x = 2x$.

Ex Rolle's thm can be used to show that a polynomial of degree n cannot have more than n roots.

$$f(x) - \frac{x-b}{a-b}(f(a) - f(b)) = g(x)$$

$$g(a) = f(b) = g(b)$$

$$g'(d) = 0$$