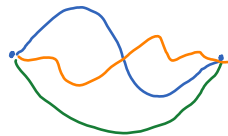


Lecture 26

Monday, November 28, 2022 6:42 AM

Questions ----

Rolle's thm: If f is cont. on $[a, b]$ and differentiable in (a, b) and $f(a) = f(b)$ then $f'(c) = 0$ for some $c \in (a, b)$.



Ex a polynomial of degree n has at most n real roots.

Ex function $x^3 + x + 1$ has exactly one real root.

Ex function $\sin x - 3x$ has exactly one real root, which is zero.

Mean-value thm:

f is cont. on $[a, b]$ and differentiable on (a, b) . Then

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

for some $c \in (a, b)$.

Proof apply Rolle's thm for $g(x) = f(x) - \frac{x-a}{b-a} [f(b) - f(a)]$

Ex $\frac{\sin x}{x} \leq 1$ for any $x \neq 0$

$$\underline{\underline{Ex}} \quad e^x \geq 1+x \quad \text{for any } x \in \mathbb{R}$$

$$\underline{\underline{Ex}} \quad \ln x < x-1 \quad \text{for any } x > -1$$