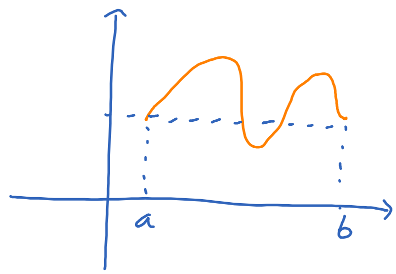
Lecture 28

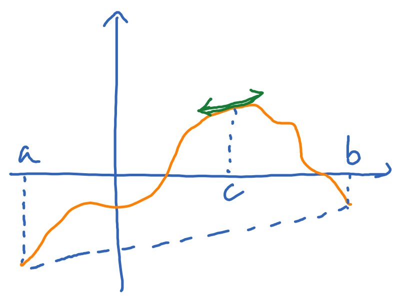
Wednesday, November 30, 2022 10:47 PM

Questions . . .



Rolle's thm requires $f(a) = f(b)$

What if $f(a) \neq f(b)$? What can we say?



Mean Value Theorem:

If f is cont. on $[a,b]$ and differentiable on (a,b) then

$$f'(c) = \frac{f(b)-f(a)}{b-a} \quad \text{for some } c \in (a,b).$$

This can be proved using Rolle's theorem for function

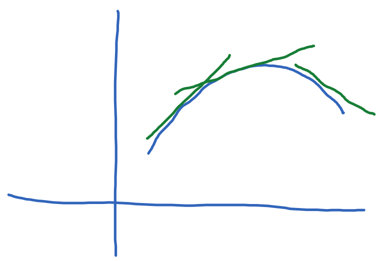
$$g(x) = f(x) - (x-a)\frac{f(b)-f(a)}{b-a}$$

Note that $g(a) = g(b) \left(= f(a)\right)$.

Ex Problem 3 on worksheet 14: show that $\sqrt[3]{x+1} < 1 + \frac{x}{3}$ for any $x > 0$.

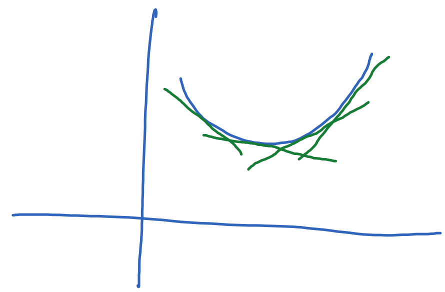
Applications of f''

$f''(x)$ is the derivative of $f'(x)$.



slope decreasing

$f''<0$



slope increasing

$f''>0$

- If $f''<0$ on an interval (a,b), we say that f is a concave function on (a,b). The graph of f is concave downward.
- If $f''>0$ on (a,b), we say that f is a convex function on (a,b). The graph of f is concave upward.
- If $f''(c)=0$, c is called an inflection point.

Ex find critical points, local min/max, inflection points of

(a): $4x^3+3x^2-6x+1$

(b): $\dfrac{x}{x^2+1}$

(c): $\dfrac{x^2}{x-1}$

$$\frac{x^2}{x-1} = x+1+\frac{1}{x-1} \rightsquigarrow f' = 1 - \frac{1}{(x-1)^2}$$

$$\rightsquigarrow f'' = \frac{2}{(x-1)^3}$$

$$4x^3 + 3x^2 - 6x + 1 \rightsquigarrow f' = 12x^2 + 6x - 6 = 6(2x^2 + x - 1)$$

$$= 6(x+1)(2x-1)$$

x	-1	$1/2$
f'	0	0
f		