Lecture 29

Friday, December 2, 2022 8:26 AM

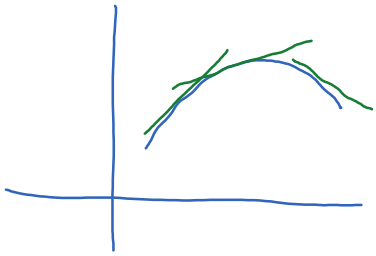
Questions...

We will use Calculus to learn some geometric behaviors of a function.

f' tells us about peak (local maximum) and valley (local minimum) and interval of increasing / decreasing of f.

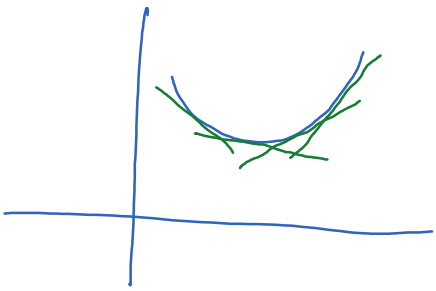
How about f''?

$f''(x)$ is the derivative of $f'(x)$.



slope decreasing

$f'' < 0$



slope increasing

$f'' > 0$

- If $f'' < 0$ on an interval (a,b), we say that f is a concave function on (a,b). The graph of f is concave downward.
- If $f'' > 0$ on (a,b), we say that f is a convex function on (a,b). The graph of f is concave upward.
- If $f''(c) = 0$, c is called an inflection point.

Ex Find critical points, local min/max, inflection points of and sketch the graph of

(a): $4x^3 + 3x^2 - 6x + 1$

(b): $\frac{x}{x^2+1}$

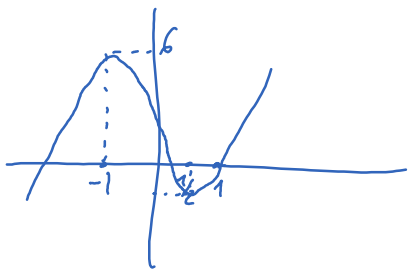
(c): $\frac{x^2}{x-1}$

(a): $f(x) = 4x^3 + 3x^2 - 6x + 1$

$f'(x) = 12x^2 + 6x - 6 = 6(2x^2 + x - 1) = 6(x+1)(2x-1)$

$f''(x) = 24x + 6 \longrightarrow$ inflection point at $x = -1/4$

x		-1		$1/2$	
f'	$+$	0	$-$	0	$+$
f	$-\infty$ ↗	6	↘	$-3/4$	↗ ∞



Some properties that one should notice:

- local min/max
- increasing/decreasing
- inflection
- domain
- asymptotes
- x, y-intercepts
- concavity
- symmetry (even/odd)