Lecture 30

Tuesday, December 6, 2022 12:57 AM

* Questions....

Graphing using Calculus:

$$f(x) = \frac{x^3}{3} - x^2 - 3x$$

* Domain: $\mathbb{R}$

* y-intercept: $(0,0)$

* x-intercept:

$$f(x) = 0 \rightsquigarrow x\left(\frac{x^2}{3} - x - 3\right) = 0$$

$$\rightsquigarrow x = 0 \quad \text{or} \quad \underbrace{\frac{x^2}{3} - x - 3 = 0}$$

$$x^2 - 3x - 9 = 0$$

$$\Delta = (-3)^2 - 4(-9) = 45 > 0$$

$$\sqrt{\Delta} = 3\sqrt{5}$$

$$x = \frac{3 \pm 3\sqrt{5}}{2}$$

$$\rightsquigarrow x = 0 \quad \text{or} \quad x \approx -1.85 \quad \text{or} \quad x \approx 4.85$$

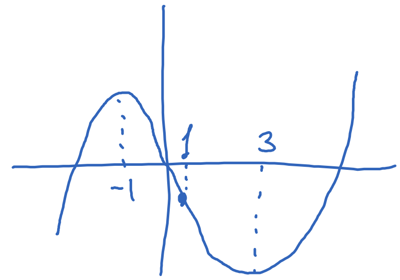
* $f'(x) = x^2 - 2x - 3 = (x+1)(x-3)$

Two roots: $x = -1$ and $x = 3$

critical numbers

Fluctuation chart :

x		-1		3	
f'	$+$	0	$-$	0	$+$
f	$-\infty \nearrow$	$5/3$	$\searrow$	-9	$\nearrow +\infty$



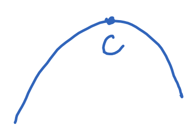
$f''(x) = 2x - 2 \rightsquigarrow f$ has only one inflection point at $x = 1$.

$\star$ Second Derivative Test :

Let c be a critical point of f. (That is, $f'(c) = 0$)

If $f''(c) > 0$ then f attains local minimum at c. In other words, $(c, f(c))$ is a valley

If $f''(c) < 0$ then f attains local maximum at c. In other words, $(c, f(c))$ is a peak.



Ex : $f(x) = \dfrac{x^3}{3} - x^2 - 3x$

has two critical numbers : $x = -1$ and $x = 3$

$f''(x) = 2x - 2$

$f''(-1) = -4 < 0$, $f''(3) = 4 > 0$. Thus, f attains local min at -1, local max at 1.