Lecture 31

Tuesday, December 6, 2022 12:57 AM

* Questions ...

Newton's method: $f(x) = 0$

$\underline{\underline{Ex}}$: find $\sqrt{2}$ $\rightsquigarrow$ $x^2 - 2 = 0$

find root of $x - \frac{1}{x^2} = 1$

find critical points of $x \sin x$

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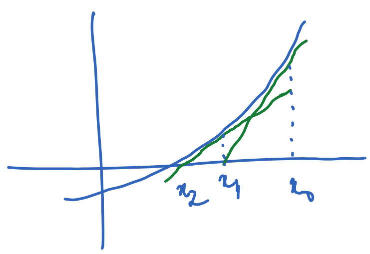


Tangent line at x_0:

$$y - f(x_0) = f'(x_0)(x - x_0)$$

x-intercept:

$$0 - f(x_0) = f'(x_0)(x - x_0)$$

$$\rightsquigarrow x = \underbrace{x_0 - \frac{f(x_0)}{f'(x_0)}}_{\text{this is } x_1}$$

Newton's method

$\underline{\text{Step 1}}$: pick a value for x_0

$\underline{\text{Step 2}}$: find $x_1, x_2, x_3,$ from the recursive formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

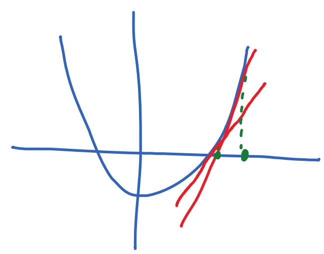
In the case $f(x) = x^2 - 2$, $f'(x) = 2x$, and

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^2 - 2}{2x_n} = x_n - \left(\frac{x_n^2}{2x_n} - \frac{2}{2x_n}\right)$$

$$= x_n - \left(\frac{x_n}{2} - \frac{1}{x_n}\right)$$

$$= \frac{x_n}{2} + \frac{1}{x_n}$$

$$\boxed{x_{n+1} = \frac{x_n}{2} + \frac{1}{x_n}}$$



Take $x_0 = 2$.

$$x_1 = \frac{x_0}{2} + \frac{1}{x_0} = \frac{2}{2} + \frac{1}{2} = 1.5$$

$$x_2 = \frac{x_1}{2} + \frac{1}{x_1} = \frac{1.5}{2} + \frac{1}{1.5} \approx 1.41666..$$

$$x_3 = \frac{x_2}{2} + \frac{1}{x_2} = \frac{1.41666.}{2} + \frac{1}{1.41666..} \approx 1.41421$$

.....

We end the recursion when certain precision has been reached, for example, when

$$|x_{n+1} - x_n| < 0.000001$$

Work on the worksheet.