Lecture 5

Monday, October 3, 2022 10:10 PM

* Question ...

Limit of a function: $\lim_{x \to a} f(x) = L$

Another notation: $f(x) \to L$ as $x \to a$

$\lim_{x \to a^-} f(x) = L$: $f(x) \to L$ as $x \to a^-$ ($x \to a$ from the left)

<u>Observation</u>: $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = L$

and $\lim_{x \to a^+} f(x) = L$.

<u>Ex</u> $\lim_{x \to 0} \frac{1}{x}$ doesn't exist

$\lim_{x \to 0} \sin \frac{1}{x}$ doesn't exist $\leftarrow$ use Maple to draw

$\lim_{x \to 0} x \sin \frac{1}{x} = 0$ $\leftarrow$ use calculator to check

<u>Ex</u> speedometer

$x(t) = t^2$

$v = \frac{\Delta x}{\Delta t}$

average velocity from time 1 and 2 is $v = \frac{x(2) - x(1)}{2 - 1} = \frac{4 - 1}{1} = 3$

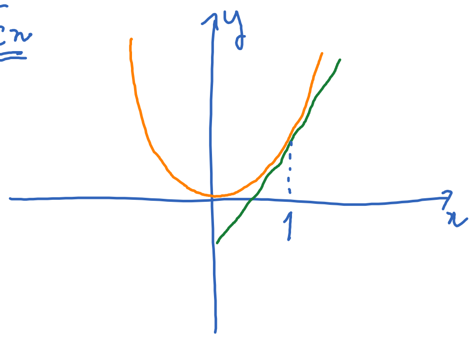
Average speed from time 1 to 1.5 is $\dfrac{x(1.5)-x(1)}{1.5-1}$.

Average speed from time 1 to $1+h$ is $\dfrac{x(1+h)-x(1)}{(1+h)-1} = \underbrace{\dfrac{x(1+h)-x(1)}{h}}_{\text{difference quotient}}$

As $h \to 0$, the average speed tends to the <u>instantenous</u> speed.

$$v_{in} = \lim_{h\to 0} \frac{(1+h)^2-1}{h} = \lim_{h\to 0} \frac{(1+2h+h^2)-1}{h} = 2$$

Ex



The slope of the tangent to the graph of $f(x)=x^2$ at $x=1$.

Ex

Guess $\displaystyle\lim_{x\to 2} \frac{\sqrt{x+2}-2}{x-2}$ $\displaystyle\lim_{t\to 0} \frac{\sin t}{\cos t}$

$\displaystyle\lim_{x\to 2} \frac{\sqrt{x+2}-2}{x+2}$ $\displaystyle\lim_{t\to 0} \frac{\cos t}{\sin t}$

$\displaystyle\lim_{t\to 0} \frac{t^2+1}{t}$