Lecture 8

Monday, October 10, 2022 7:51 AM

Limit laws:

* Plug in (for continuous function)
* Addition law
* Multiplication/quotient law
* Composition law

If $\lim_{x \to a} f(x) = b$, $\lim_{x \to b} g(x) = c$

then $\lim_{x \to a} f(g(x)) = c$

Ex $\lim_{x \to 0} \cos^4 x$

$x \xrightarrow[g]{\text{cosine}} \cos x \xrightarrow[f]{\wedge 4} \cos^4 x$

As $x \to 0$, $\cos x \to \cos 0 = 1$, $\cos^4 x \to \cos^4 0 = 1^4 = 1$.

* Squeeze theorem

If $f(x) \leq g(x) \quad \forall x$

then $\lim_{x \to a} f(x) \leq \lim_{x \to a} g(x)$

If $\begin{cases} g(x) \le f(x) \le h(x) & \forall x \text{ near } a \\ \lim_{x\to a} g(x) = \lim_{x\to a} h(x) = L \end{cases}$

Then $\lim_{x\to a} f(x) = L$.

Ex: $\lim_{x\to 0} x \sin\frac{1}{x}$

$\lim_{x\to 0} x \sin\frac{1}{x^2}$