Lecture 9

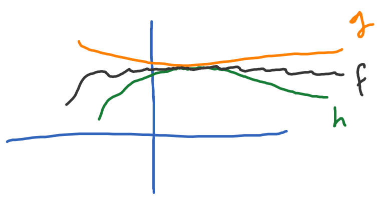
Tuesday, October 11, 2022 4:39 AM

* Question

Squeeze theorem/law:

If $g(x) \le f(x) \le h(x)$ when $x \approx a$ and $\lim_{x\to a} g(x) = \lim_{x \to a} h(x) = L$ then

$$\lim_{x\to a} f(x) = L.$$



Ex $\lim_{x\to 0} \Big[\underbrace{e^x + x\sin\left(e^{\frac{1}{x^2}}\right)}_{f(x)}\Big]$

$$-|x| \le x\sin\left(e^{\frac{1}{x^2}}\right) \le |x|$$

$x \to 0$

0

$$\underbrace{e^x - |x|}_{g(x)} \le f(x) \le \underbrace{e^x + |x|}_{h(x)}$$

$x \to 0$

1

Thus, $\lim_{x\to 0} f(x) = 1$. worksheet problem

Property: (proof in the textbook)

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

Ex

$$\lim_{x\to 0} \frac{\sin^2 3x}{2x^2} = \lim_{x\to 0} \left(\frac{\sin 3x}{3x}\right)^2 \frac{9}{2} = \frac{9}{2}\Big(\underbrace{\lim_{x\to 0} \frac{\sin 3x}{3x}}_{=1}\Big)^2 = \frac{9}{2}$$

Ex

$$\lim_{x\to 0} \frac{\sin x}{x+\sin 2x} = \lim_{x\to 0} \frac{\frac{\sin x}{x}}{\frac{x}{x}+\frac{\sin 2x}{x}} = \lim_{x\to 0} \frac{\frac{\sin x}{x}}{1+2\frac{\sin 2x}{2x}}$$

$$\underset{\text{quotient law}}{=} \frac{\lim \frac{\sin x}{x}}{\lim\left(1+2\frac{\sin 2x}{2x}\right)} \underset{\text{add. law}}{=} \frac{1}{1+2\lim \frac{\sin 2x}{2x}} = \frac{1}{3}$$

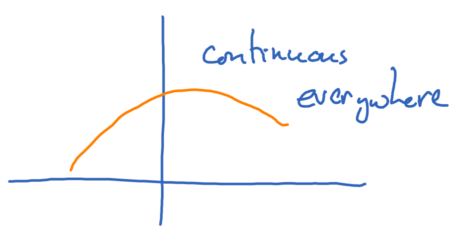
worksheet problems

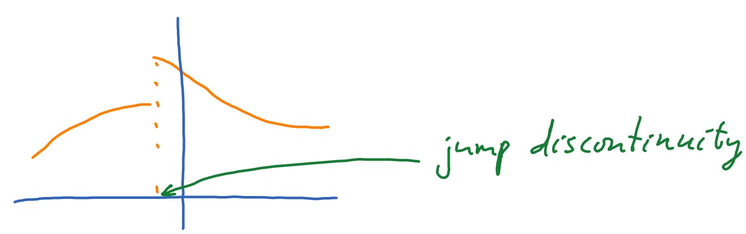
Continuous functions

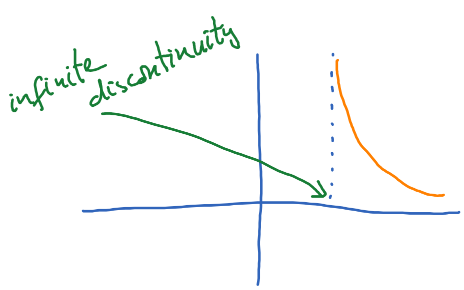
f is continuous at a if $\lim_{x\to a} f(x) = f(a)$.

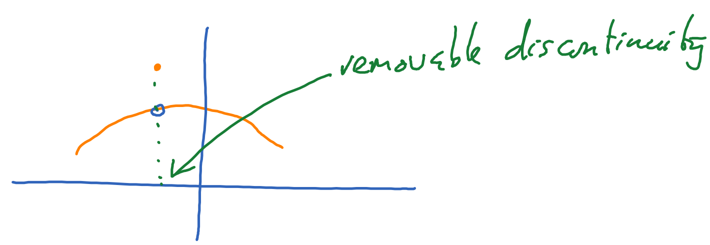
f is continuous on a set A if f is continuous at all $a \in A$.

Ex









If f is continuous on $[a,b]$, we can draw f with one stroke on $[a,b]$.

Another way:

$$\lim_{x \to a} f(x) = f\left(\lim_{x \to a} x\right) \qquad \text{"}f \text{ interchangeable with limit"}$$